ETS PRAXIS

Measuring the Power of Learning.™

The *Praxis*® Study Companion

Mathematics: Content Knowledge

5161

www.ets.org/praxis

Welcome to the *Praxis*® Study Companion

Prepare to Show What You Know

You have been working to acquire the knowledge and skills you need for your teaching career. Now you are ready to demonstrate your abilities by taking a *Praxis*® test.

Using the *Praxis*® *Study Companion* is a smart way to prepare for the test so you can do your best on test day. This guide can help keep you on track and make the most efficient use of your study time.

The Study Companion contains practical information and helpful tools, including:

- An overview of the *Praxis* tests
- Specific information on the *Praxis* test you are taking
- A template study plan
- Study topics
- Practice questions and explanations of correct answers
- Test-taking tips and strategies
- Frequently asked questions
- Links to more detailed information

So where should you start? Begin by reviewing this guide in its entirety and note those sections that you need to revisit. Then you can create your own personalized study plan and schedule based on your individual needs and how much time you have before test day.

Keep in mind that study habits are individual. There are many different ways to successfully prepare for your test. Some people study better on their own, while others prefer a group dynamic. You may have more energy early in the day, but another test taker may concentrate better in the evening. So use this guide to develop the approach that works best for you.

Your teaching career begins with preparation. Good luck!

Know What to Expect

Which tests should I take?

Each state or agency that uses the *Praxis* tests sets its own requirements for which test or tests you must take for the teaching area you wish to pursue.

Before you register for a test, confirm your state or agency's testing requirements at www.ets.org/praxis/states.

How are the *Praxis* tests given?

Praxis tests are given on computer. Other formats are available for test takers approved for accommodations (see page 60).

What should I expect when taking the test on computer?

When taking the test on computer, you can expect to be asked to provide proper identification at the test center. Once admitted, you will be given the opportunity to learn how the computer interface works (how to answer questions, how to skip questions, how to go back to questions you skipped, etc.) before the testing time begins. Watch the What to Expect on Test Day video to see what the experience is like.

Where and when are the *Praxis* tests offered?

You can select the test center that is most convenient for you. The *Praxis* tests are administered through an international network of test centers, which includes Prometric® Testing Centers, some universities, and other locations throughout the world.

Testing schedules may differ, so see the *Praxis* web site for more detailed test registration information at www.ets.org/praxis/register.

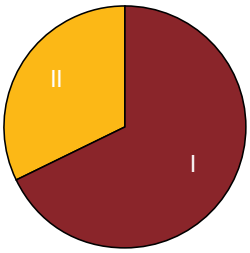
Table of Contents

The Praxis® Study Companion guides you through the steps to success

1. Learn About Your Test

Learn about the specific test you will be taking

Mathematics: Content Knowledge (5161)

Test at a Glance			
Test Name	Mathematics: Content Knowledge		
Test Code	5161		
Time	150 minutes		
Number of Questions	60		
Format	Selected-response questions–select one answer; selected-response questions–select one or more answers; numeric entry questions; drag-and-drop questions; text completion questions; and other innovative question types. On-screen graphing calculator provided		
Test Delivery	Computer delivered		
	**Content Categories**	**Approximate Number of Questions**	**Approximate Percentage of Examination**
	I. Number and Quantity, Algebra, Functions, and Calculus	41	68%
	II. Geometry, Probability and Statistics, and Discrete Mathematics	19	32%

About This Test

The *Praxis* Mathematics Content Knowledge test is designed to assess the mathematical knowledge and competencies necessary for a beginning teacher of secondary school mathematics. Examinees have typically completed a bachelor's program with an emphasis in mathematics or mathematics education.

The examinee will be required to understand and work with mathematical concepts, to reason mathematically, to make conjectures, to see patterns, to justify statements using informal logical arguments, and to construct simple proofs. Additionally, the examinee will be expected to solve problems by integrating knowledge from different areas of mathematics, to use various representations of concepts, to solve problems that have several solution paths, and to develop mathematical models and use them to solve real-world problems.

The test is not designed to be aligned with any particular school mathematics curriculum, but it is intended to be consistent with the recommendations of national studies on mathematics education, such as the National Governors Association Center for Best Practices and the Council of Chief State School Officers *Common Core State Standards in Mathematics* (2010), the National Council of Teachers of Mathematics (NCTM) and the Council for the Accreditation of Educator Preparation (CAEP) *NCTM CAEP Standards* (2012), and the NCTM *Principles and Standards for School Mathematics* (2000).

This test may contain some questions that will not count toward your score.

On-Screen Graphing Calculator

An on-screen graphing calculator is provided for the computer-delivered test. Please consult the Praxis Calculator Use web page for further information.

You are expected to know how and when to use the graphing calculator since it will be helpful for some questions. You are expected to become familiar with its functionality before taking the test. To practice using the calculator, download the 30-day trial version and view tutorials on how to use it. The calculator may be used to perform calculations (e.g., exponents, roots, trigonometric values, logarithms), to graph and analyze functions, to find numerical solutions to equations, and to generate a table of values for a function.

Using Your Calculator

Take time to download the 30-day trial version of the calculator. View the tutorials on the website. Practice with the calculator so that you are comfortable using it on the test.

There are only some questions on the test for which a calculator is helpful or necessary. First, decide how you will solve a problem, then determine if you need a calculator. For many questions, there is more than one way to solve the problem. Don't use the calculator if you don't need to; you may waste time.

Sometimes answer choices are rounded, so the answer that you get might not match the answer choices in the question. Since the answer choices are rounded, plugging the choices into the question might not produce an exact answer.

Don't round any intermediate calculations. For example, if the calculator produces a result for the first step of a solution, keep the result in the calculator and use it for the second step. If you round the result from the first step and the answer choices are close to each other, you might choose the incorrect answer.

Read the question carefully so that you know what you are being asked to do. Sometimes a result from the calculator is NOT the final answer. If an answer you get is not one of the choices in the question, it may be that you didn't answer the question being asked. Read the question again. It might also be that you rounded at an intermediate step in solving the problem.

Think about how you are going to solve the question before using the calculator. You may only need the calculator in the final step or two. Don't use it more than necessary.

Check the calculator modes (degree versus radian, floating decimal versus scientific notation) to see that these are correct for the question being asked.

Make sure that you know how to perform the basic arithmetic operations and calculations (e.g., exponents, roots, trigonometric values, logarithms). Your test may involve questions that require you to do some of the following: graph functions and analyze the graphs, find zeros of functions, find points of intersection of graphs of functions, find minima/maxima of functions, find numerical solutions to equations, and generate a table of values for a function.

Test Specifications

Test specifications describe the knowledge and skills measured by the test. Study topics that help you prepare to answer test questions can be found on page 42. Because the assessment was designed to measure the ability to integrate knowledge of mathematics, answering any question may involve more than one competency and may involve competencies from more than one content category.

I. Number and Quantity, Algebra, Functions, and Calculus

A. Number and Quantity

1. Understand the properties of exponents
   a. perform operations involving exponents, including negative and rational exponents
   b. demonstrate an understanding of the properties of exponential expressions
   c. use the properties of exponents to rewrite expressions that have radicals or rational exponents
2. Understand the properties of rational and irrational numbers, and the interactions between those sets of numbers
   a. recognize that the sum or product of two rational numbers is rational
   b. recognize that the sum of a rational number and an irrational number is irrational
   c. recognize that the product of a nonzero rational number and an irrational number is irrational
   d. recognize that the sum or product of two irrational numbers can be rational or irrational
3. Understand how to solve problems by reasoning quantitatively (e.g., dimensional analysis, reasonableness of solutions)
   a. use units as a way to understand problems and to guide the solution of multistep problems
   b. choose and interpret units consistently in formulas
   c. choose and interpret the scale and the origin in graphs and data displays
   d. recognize the reasonableness of results within the context of a given problem
4. Understand the structure of the natural, integer, rational, real, and complex number systems and how the basic operations (+, –, ×, and ÷) on numbers in these systems are performed
   a. solve problems using addition, subtraction, multiplication, and division of rational, irrational, and complex numbers
   b. apply the order of operations
   c. given operations on a number system, determine whether the properties (e.g., commutative, associative, distributive) hold
   d. compare, classify, and order real numbers
   e. simplify and approximate radicals
   f. find conjugates of complex numbers
   g. demonstrate an understanding of the properties of counting numbers (e.g., prime, composite, prime factorization, even, odd, factors, multiples)
5. Understand how to work with complex numbers when solving polynomial equations and rewriting polynomial expressions
   a. solve quadratic equations with real coefficients that have complex solutions
   b. extend polynomial identities to the complex numbers (e.g., $x^2 + y^2 = (x + yi)(x - yi)$)
   c. verify the fundamental theorem of algebra for quadratic polynomials
6. Understand how to perform operations on matrices and how to use matrices in applications
   a. use matrices to represent and manipulate data
   b. multiply matrices by scalars to produce new matrices
   c. add, subtract, and multiply matrices of appropriate dimensions
   d. understand that matrix multiplication for square matrices is not a commutative operation but still satisfies the associative and distributive properties
   e. understand the role played by zero and identity matrices in matrix addition and multiplication
   f. understand that the determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse
   g. work with 2 × 2 matrices as transformations

of the plane and interpret the absolute value of the determinant in terms of area

7. Understand how to solve problems involving ratios, proportions, averages, percents, and metric and traditional unit conversions
   a. apply the concept of a ratio and use ratio language and notation to describe a relationship between two quantities
   b. compute unit rates
   c. use ratio reasoning to convert rates
   d. solve problems involving scale factors
   e. recognize and represent proportional and inversely proportional relationships between two quantities
   f. use proportional relationships to solve multistep ratio, average, and percent problems
   g. solve measurement and estimation problems involving time, length, temperature, volume, and mass in both the U.S. customary system and the metric system, where appropriate
   h. convert units within the metric and U.S. customary systems
8. Know how to analyze both precision and accuracy in measurement situations
   a. choose a level of accuracy appropriate to limitations on measurement when reporting quantities
   b. calculate or estimate absolute and relative error in the numerical answer to a problem
9. Understand various ways to represent and compare very large and very small numbers (e.g., scientific notation, orders of magnitude)
   a. represent and compare very large and very small numbers
10. Understand how to both estimate and perform calculations on very large and very small quantities
    a. use orders of magnitude to estimate very large and very small numbers
    b. perform calculations on numbers in scientific notation

B. Algebra

1. Understand how to write algebraic expressions in equivalent forms
   a. use the structure of an expression to identify ways to rewrite it
   b. understand how to rewrite quadratic expressions for specific purposes (e.g., factoring/finding zeros, completing the square/finding maxima or minima)
   c. use the properties of exponents to rewrite expressions for exponential functions
2. Understand how to perform arithmetic operations on polynomials
   a. add, subtract, and multiply polynomials
3. Understand the relationship between zeros of polynomial functions (including their graphical representation) and factors of the related polynomial expressions
   a. know and apply the remainder theorem: for a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$
   b. use factorization to identify zeros of polynomials
   c. use zeros of a polynomial to construct a rough graph of the function defined by the polynomial
4. Understand how to use polynomial identities (e.g., difference of squares, sum and difference of cubes) to solve problems
   a. apply the binomial theorem for the expansion of $(x+y)^n$ in powers of x and y for a positive integer n
5. Understand how to rewrite rational expressions and perform arithmetic operations on rational expressions
   a. rewrite simple rational expressions in different forms
   b. understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression
   c. add, subtract, multiply, and divide rational expressions

6. Understand how to create equations and inequalities that describe relationships
   a. create equations and inequalities in one variable and use them to solve problems and graph solutions on the number line
   b. create equations and inequalities in two or more variables to represent relationships between quantities, solve problems, and graph them on the coordinate plane with labels and scales
   c. represent constraints by equations, inequalities, or systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context
   d. rearrange formulas to highlight a quantity of interest (e.g., solve $d = rt$ for t)
7. Understand how to justify the reasoning process used to solve equations, including analysis of potential extraneous solutions
   a. explain each step in solving a simple equation
   b. solve simple rational and radical equations in one variable, incorporating analysis of possible extraneous solutions
8. Understand how varied techniques (e.g., graphical, algebraic) are used to solve equations and inequalities in one variable
   a. solve linear equations and inequalities in one variable, including equations with coefficients represented by letters
   b. use the method of completing the square to transform any quadratic equation in x into the equivalent form $(x - p)^2 = q$.
   c. solve equations using a variety of methods (e.g., using graphs, using the quadratic formula, factoring)
   d. use different methods (e.g., discriminant analysis, graphical analysis) to determine the nature of the solutions of a quadratic equation
   e. write complex solutions in the form $a \pm bi$.
9. Understand how varied techniques (e.g., graphical, algebraic, matrix) are used to solve systems of equations and inequalities
   a. explain why, when solving a system of two equations using the elimination method, replacing one or both equations with a scalar multiple produces a system with the same solutions as the solutions of the original system
   b. solve a system consisting of two linear equations in two variables algebraically and graphically
   c. solve a system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically
   d. represent a system of linear equations as a single matrix equation
   e. find the inverse of a matrix, if it exists, and use it to solve systems of linear equations
   f. explain why the x-coordinates of the intersection points of the graphs of $y = f(x)$ and $y = g(x)$ are the solutions of $f(x) = g(x)$
   g. find the solutions of $f(x) = g(x)$ approximately (e.g., use technology to graph the functions, make tables of values, find successive approximations); include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, or logarithmic functions
   h. graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes
10. Understand the properties of number systems under various operations
    a. given operations on algebraic expressions, determine whether the properties hold (e.g., commutative, associative, distributive)
11. Understand the concept of rate of change of nonlinear functions
    a. calculate and interpret the average rate of change of a function presented symbolically, numerically, or graphically over a specified interval

12. Understand the concepts of intercept(s) of a line and slope as a rate of change
    a. calculate and interpret the intercepts of a line
    b. calculate and interpret the slope of a line presented symbolically, numerically, or graphically
    c. estimate the rate of change of a linear function from a graph
13. Understand how to find the zero(s) of functions
    a. use a variety of techniques to find and analyze the zero(s) (real and complex) of functions

C. Functions

1. Understand the function concept and the use of function notation
    a. recognize that functions are sets of ordered pairs
    b. understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range
    c. use function notation, evaluate functions, and interpret statements that use function notation in terms of a context
    d. recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers
2. Understand how to find the domain and range of a function and a relation
    a. identify the domain and range of a function or relation
    b. determine the domain of a function from a function rule (e.g., $f(x) = 2x + 1$), graph, set of ordered pairs, or table
3. Understand how function behavior is analyzed using different representations (e.g., graphs, mappings, tables)
    a. interpret key features of graphs and tables (e.g., increasing/decreasing, maximum/minimum, periodicity) in terms of the quantities for a function that models a relationship between two quantities
    b. given a verbal description of a relation, sketch graphs that show key features of that relation
    c. graph functions (i.e., radical, piecewise, absolute value, polynomial, rational, logarithmic, trigonometric) expressed symbolically and identify key features of the graph
    d. write a function that is defined by an expression in different but equivalent forms to reveal different properties of the function (e.g., zeros, extreme values, symmetry of the graph)
    e. interpret the behavior of exponential functions (e.g., growth, decay)
    f. understand how to determine if a function is odd, even, or neither, and any resulting symmetries
4. Understand how functions and relations are used to model relationships between quantities
    a. write a function that relates two quantities
    b. determine an explicit expression or a recursive process that builds a function from a context
5. Understand how new functions are obtained from existing functions (e.g., compositions, transformations, inverses)
    a. describe how the graph of $g(x)$ is related to the graph of $f(x)$, where $g(x) = f(x) + k$, $g(x) = k\,f(x)$, $g(x) = f(kx)$, or $g(x) = f(x + k)$ for specific values of k (both positive and negative), and find the value of k given the graphs
    b. determine if a function has an inverse and, if so, write an expression for the inverse
    c. verify by composition if one function is the inverse of another
    d. given that a function f has an inverse, find values of the inverse function from a graph or a table of f
    e. given a noninvertible function, determine a largest possible domain of the function that produces an invertible function
    f. understand the inverse relationship between exponential and logarithmic functions and use this relationship to solve problems
    g. combine standard function types using arithmetic operations
    h. perform domain analysis on functions resulting from arithmetic operations

i. compose functions algebraically, numerically, and graphically

j. perform domain analysis on functions resulting from compositions

6. Understand differences between linear, quadratic, and exponential models, including how their equations are created and used to solve problems

a. understand that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals

b. recognize situations in which one quantity changes at a constant rate per unit interval relative to another

c. recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another

d. construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two ordered pairs (including reading these from a table)

e. observe that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function

f. express the solution to an exponential equation with base *b* as a logarithm (e.g., $3 \cdot 2^{5t} = 20,\ 3 \cdot e^{5t} = 20$)

g. use technology to evaluate logarithms that have any base

h. interpret the parameters in a linear or exponential function in terms of a context (e.g., $A(t) = Pe^{rt}$)

i. use quantities that are inversely related to model phenomena

7. Understand how to construct the unit circle and how to use it to find values of trigonometric functions for all angle measures in their domains

a. understand radian measure (e.g., one radian is the measure of a central angle that subtends an arc with length equal to the length of the radius)

b. understand how the domains of trigonometric functions can be extended beyond 0 to 2π using the unit circle

c. use special triangles (i.e., 30-60-90, 45-45-90) to determine geometrically the values of sine, cosine, and tangent for $\frac{\pi}{3}$, $\frac{\pi}{4}$, and $\frac{\pi}{6}$

d. use reference angles to find the values of trigonometric functions at angles outside the interval 0 to $\frac{\pi}{2}$

e. use the unit circle to explain symmetry and periodicity of trigonometric functions

8. Understand how periodic phenomena are modeled using trigonometric functions

a. choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline

b. understand how to restrict the domain of a trigonometric function so that its inverse can be constructed

c. use inverse functions to solve trigonometric equations that arise in modeling contexts, and interpret them in terms of the context

9. Understand the application of trigonometric identities (e.g., Pythagorean, double angle, half angle, sum of angles, difference of angles)

a. use Pythagorean identities (e.g., $\sin^2\theta + \cos^2\theta = 1$)

b. use trigonometric identities to rewrite expressions and solve equations

c. understand trigonometric identities in the context of equivalent graphs of trigonometric functions (e.g., $y = \sin x$ and $y = \cos\left(\frac{\pi}{2} - x\right)$ are equivalent graphs)

d. prove Pythagorean identities (e.g., $\sin^2\theta + \cos^2\theta = 1$)

10. Know how to interpret representations of functions of two variables (e.g., three-dimensional graphs, tables)

a. interpret representations of functions of two variables

11. Understand how to solve equations (e.g., trigonometric, logarithmic, exponential)

a. solve trigonometric, logarithmic, and exponential equations

D. Calculus

1. Understand the meaning of a limit of a function and how to calculate limits of functions, determine when the limit does not exist, and solve problems using the properties of limits
   a. graphically analyze the limit of $f(x)$ as x approaches a fixed value from both left and right
   b. solve limit problems (e.g., a constant times a function, the sum of two functions, the product and quotient of two functions) using properties of limits, where all limits of the individual functions exist at the value that x is approaching
   c. analyze one-sided limits for various functions to see whether or not the limit exists
   d. recognize limits that do not exist, such as

$$\lim_{x\to 0} \sin\left(\frac{1}{x}\right) \text{ and } \lim_{x\to 0} \frac{1}{\sqrt[3]{x^2}}$$

2. Understand the derivative of a function as a limit, as the slope of a line tangent to a curve, and as a rate of change
   a. construct a function graph for a given function and a given point $(a, f(a))$, and explain what happens to the succession of slopes of secant lines connecting $(a, f(a))$ to $(x, f(x))$ as x approaches a, from both the right side and the left side
   b. state the limit definition of the derivative, and use it to find the derivative of a given function at a given value of x and to find the derivative function
3. Understand how to show that a particular function is continuous
   a. apply the three steps (i.e., $f(a)$ exists, $\lim_{x\to a} f(x)$ exists, and $f(a) = \lim_{x\to a} f(x)$) that are part of the definition of what it means for a function to be continuous at $x = a$ to verify whether a given function is continuous at a given point
4. Know the relationship between continuity and differentiability
   a. give examples of functions that are continuous at $x = a$ but not differentiable at $x = a$, and explain why
5. Understand how to approximate derivatives and integrals numerically
   a. given a table of values, use the slope of a secant line to approximate a derivative
   b. use the midpoint rule, trapezoid rule, or other Reimann sums to find numerical approximations for integrals
6. Understand how and when to use standard differentiation and integration techniques
   a. use standard differentiation techniques
   b. use standard integration techniques
   c. understand the relationship between position, velocity, and acceleration functions of a particle in motion
7. Understand how to analyze the behavior of a function (e.g., extrema, concavity, symmetry)
   a. use the first and second derivatives to analyze the graph of a function
8. Understand how to apply derivatives to solve problems (e.g., related rates, optimization)
   a. apply derivatives to solve problems
9. Understand the foundational theorems of calculus (e.g., fundamental theorems of calculus, mean value theorem, intermediate value theorem)
   a. solve problems using the foundational theorems of calculus
   b. understand the relationship between differentiation and integration, including the role of the fundamental theorems of calculus
   c. match graphs of functions with graphs of their derivatives or accumulations
   d. understand how to use differentiation and integration of a function to express rates of change and total change
   e. understand and calculate the average value of a function over an interval (i.e., mean value theorem of integrals)
10. Understand integration as a limit of Riemann sums
    a. calculate a definite integral using a limit of Riemann sums

11. Understand how to use integration to compute area, volume, distance, or other accumulation processes
    a. use integration techniques to compute area, volume, distance, or other accumulation processes
12. Know how to determine the limits of sequences, if they exist
    a. determine the limits of sequences, when they exist
13. Is familiar with simple infinite series
    a. determine if simple infinite series converge or diverge
    b. find the sum of a simple infinite series, if it exists
    c. find the partial sum of a simple infinite series

II. Geometry, Probability and Statistics, and Discrete Mathematics

A. Geometry

1. Understand transformations in a plane
    a. know precise definitions of angle, circle, line segment, perpendicular lines, and parallel lines
    b. represent transformations in the plane
    c. describe transformations as functions that take points in the plane as inputs, and give other points as outputs
    d. recognize whether a transformation preserves distance and angle measure
    e. given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that map it onto itself
    f. develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments
    g. given a geometric figure and a rotation, reflection, or translation, draw the transformed figure
    h. specify a sequence of transformations that will map a given figure onto another figure
2. Understand how to prove geometric theorems, such as those about lines and angles, triangles, and parallelograms
    a. prove theorems about lines and angles
    b. prove theorems about triangles
    c. prove theorems about parallelograms
3. Understand how geometric constructions are made with a variety of tools and methods
    a. recognize formal geometric constructions
    b. explain how formal geometric constructions are made (e.g., an equilateral triangle, a square, a regular hexagon inscribed in a circle)
4. Understand congruence and similarity in terms of transformations
    a. use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure
    b. verify the properties of dilations given by a center and a scale factor
    c. given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent
    d. given two figures, use the definition of similarity in terms of dilations to decide if the figures are similar
    e. explain how the criteria for triangle congruence (e.g., ASA, SAS, HL) follow from the definition of congruence in terms of rigid motions
    f. use the properties of similarity transformations to establish the AA criterion for two triangles to be similar
    g. use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures
5. Understand how trigonometric ratios are defined in right triangles
    a. understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles
    b. explain and use the relationship between the sine and cosine of complementary angles
    c. use trigonometric ratios and the Pythagorean theorem to solve right triangles in applied problems

6. Understand how trigonometry is applied to general triangles
   a. derive the formula $A = \frac{1}{2}ab\sin C$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side and use it to solve problems
   b. apply the law of sines and the law of cosines to find unknown measurements in triangles
7. Understand and applies theorems about circles
   a. identify and describe relationships among inscribed angles, radii, and chords
   b. prove properties of angles for a quadrilateral inscribed in a circle
   c. construct a tangent line from a point outside a given circle to the circle
8. Understand arc length and area measurements of sectors of circles
   a. derive and use the fact that the length of the arc intercepted by a central angle is proportional to the circumference
   b. derive and use the formula for the area of a sector
9. Know how to translate between a geometric description (e.g., focus, asymptotes, directrix) and an equation for a conic section
   a. derive and use the equation of a circle of given center and radius
   b. complete the square to find the center and radius of a circle given by an equation in standard form
   c. derive the equation of a parabola given a focus and directrix
   d. derive and use the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from a point on the curve to the foci is constant
10. Understand how to use coordinate geometry to algebraically prove simple geometric theorems
    a. use coordinates to prove simple geometric theorems algebraically
    b. prove the slope criteria for parallel and perpendicular lines, and use parallel and perpendicular lines to solve geometric problems
    c. find the point on a directed line segment between two given points that partitions the segment in a given ratio
    d. use coordinates to compute perimeters of polygons and areas of triangles and quadrilaterals
11. Understand how perimeter, area, surface area, and volume formulas are used to solve problems
    a. give an informal argument for the formulas for the circumference of a circle, the area of a circle, and the volume of a cylinder, pyramid, and cone
    b. use the perimeter and area of geometric shapes to solve problems
    c. use the surface area and volume of prisms, cylinders, pyramids, cones, and spheres to solve problems
12. Know how to visualize relationships (e.g., cross section, nets, rotations) between two-dimensional and three-dimensional objects
    a. identify the shapes of two-dimensional cross sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects
    b. use two-dimensional representations of three-dimensional objects to visualize and solve problems
13. Know how to apply geometric concepts in real-world situations
    a. use geometric shapes, their measures, and their properties to describe objects
    b. apply concepts of density based on area and volume in modeling situations
    c. apply geometric methods to solve design problems
14. Understand the properties of parallel and perpendicular lines, triangles, quadrilaterals, polygons, and circles and how they can be used in problem solving
    a. solve problems involving parallel, perpendicular, and intersecting lines
    b. apply angle relationships (e.g., supplementary, vertical, alternate interior) to solve problems
    c. solve problems that involve medians, midpoints, and altitudes

d. solve problems involving special triangles (e.g., isosceles, equilateral, right)

e. know geometric properties of various quadrilaterals (e.g., parallelograms, trapezoids)

f. know relationships among quadrilaterals

g. solve problems involving angles and diagonals

h. solve problems involving polygons with more than four sides

B. Probability and Statistics

1. Understand how to summarize, represent, and interpret data collected from measurements on a single variable (e.g., box plots, dot plots, normal distributions)
   a. represent data with plots on the real number line (e.g., dot plots, histograms, and box plots)
   b. use statistics appropriate to the shape of the data distribution to compare center (e.g., median, mean) and spread (e.g., interquartile range, standard deviation) of two or more different data sets
   c. interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of outliers
   d. use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages, and recognize that there are data sets for which such a procedure is not appropriate
   e. estimate areas under the normal curve
2. Understand how to summarize, represent, and interpret data collected from measurements on two variables, either categorical or quantitative (e.g., scatterplots, time series)
   a. summarize and interpret categorical data for two categories in two-way frequency tables (e.g., joint, marginal, conditional relative frequencies)
   b. recognize possible associations and trends in the data
   c. represent data for two quantitative variables on a scatterplot, and describe how the variables are related
3. Understand how to create and interpret linear regression models (e.g., rate of change, intercepts, correlation coefficient)
   a. use technology to fit a function to data (i.e., linear regression)
   b. use functions fitted to data to solve problems in the context of the data
   c. assess the fit of a function by plotting and analyzing residuals
   d. interpret the slope and the intercept of a regression line in the context of the data
   e. compute and interpret a correlation coefficient
   f. distinguish between correlation and causation
4. Understand statistical processes and how to evaluate them
   a. understand statistics as a process for making inferences about population parameters based on a random sample from that population
   b. decide if a specified model is consistent with results from a given data-generating process (e.g., using simulation)
5. Understand how to make inferences and justify conclusions from samples, experiments, and observational studies
   a. recognize the purposes of and differences among samples, experiments, and observational studies, and explain how randomization relates to each
   b. use data from a sample to estimate a population mean or proportion
   c. use data from a randomized experiment to compare two treatments
   d. use results of simulations to decide if differences between parameters are significant
   e. evaluate reports based on data
6. Understand the concepts of independence and conditional probability and how to apply these concepts to data
   a. describe events as subsets of a sample space using characteristics of the outcomes, or as unions, intersections, or complements of other events

b. understand that two events, *A* and *B*, are independent if and only if $P(A \cap B) = P(A)P(B)$

c. understand the conditional probability of *A* given *B* as $\frac{P(A \text{ and } B)}{P(B)}$, and interpret independence of *A* and *B* as saying that $P(A|B) = P(A)$ and $P(B|A) = P(B)$

d. recognize and explain the concepts of conditional probability and independence

7. Understand how to compute probabilities of simple events, probabilities of compound events, and conditional probabilities

a. calculate probabilities of simple and compound events

b. construct and interpret two-way frequency tables of data when two categories are associated with each object being classified; use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities

c. find $P(A|B)$, and interpret it in terms of a given model

d. apply the addition rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret it in terms of a given model

e. apply the general multiplication rule in a uniform probability model, $P(A \text{ and } B) = P(A)P(B|A) = P(B)P(A|B)$, and interpret it in terms of a given model

f. calculate probabilities using the binomial probability distribution

8. Know how to make informed decisions using probabilities and expected values

a. define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space, and graph the corresponding probability distribution using the same graphical displays as for data distributions

b. calculate the expected value of a random variable, and interpret it as the mean of the probability distribution

c. develop a probability distribution for a random variable, defined for a sample space in which theoretical probabilities can be calculated, and find the expected value

d. develop a probability distribution for a random variable, defined for a sample space in which probabilities are assigned empirically, and find the expected value

e. weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values

f. analyze decisions and strategies using probability concepts (e.g., fairness)

9. Understand how to use simulations to construct experimental probability distributions and to make informal inferences about theoretical probability distributions

a. given the results of simulations, construct experimental probability distributions

b. given the results of simulations, make informal inferences about theoretical probability distributions

10. Understand how to find probabilities involving finite sample spaces and independent trials

a. use the fundamental counting principle to find probabilities involving finite sample spaces and independent trials

C. Discrete Mathematics

1. Understand sequences (e.g., arithmetic, recursively defined, geometric)

a. write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms

b. evaluate, extend, or algebraically represent rules that involve number patterns

c. explore patterns in order to make conjectures, predictions, or generalizations

2. Is familiar with how recursion can be used to model various phenomena
    a. find values of functions defined recursively, and understand how recursion can be used to model various phenomena
    b. convert between recursive and closed-form expressions for a function, where possible
3. Has knowledge of equivalence relations
    a. determine whether a binary relation on a set is reflexive, symmetric, or transitive
    b. determine whether a relation is an equivalence relation
4. Understand the differences between discrete and continuous representations (e.g., data, functions) and how each can be used to model various phenomena
    a. understand the differences between discrete and continuous representations (e.g., data, functions)
    b. understand how discrete and continuous representations can be used to model various phenomena
5. Understand basic terminology and symbols of logic
    a. understand the basic terminology of logic
    b. understand the symbols of logic
    c. use logic to evaluate the truth of statements
    d. use logic to evaluate the equivalence of statements (e.g., statement and contrapositive)
6. Understand how to use counting techniques such as the multiplication principle, permutations, and combinations
    a. use counting techniques to solve problems
7. Understand basic set theory (e.g., unions, differences, Venn diagrams)
    a. solve problems using basic set theory (i.e., union, intersection, complement, difference)
    b. use Venn diagrams to answer questions about sets

2. Familiarize Yourself with Test Questions

Become comfortable with the types of questions you'll find on the Praxis *tests*

The *Praxis* assessments include a variety of question types: constructed response (for which you write a response of your own); selected response, for which you select one or more answers from a list of choices or make another kind of selection (e.g., by clicking on a sentence in a text or by clicking on part of a graphic); and numeric entry, for which you enter a numeric value in an answer field. You may be familiar with these question formats from taking other standardized tests. If not, familiarize yourself with them so you don't spend time during the test figuring out how to answer them.

Understanding Computer-Delivered Questions

Questions on computer-delivered tests are interactive in the sense that you answer by selecting an option or entering text on the screen. If you see a format you are not familiar with, read the directions carefully. The directions always give clear instructions on how you are expected to respond.

For most questions, you respond by clicking an oval to select a single answer from a list of answer choices.

However, interactive question types may also ask you to respond by:

- **Clicking more than one oval** to select answers from a list of choices.
- **Typing in an entry box.** When the answer is a number, you may be asked to enter a numerical answer. Some questions may have more than one place to enter a response.
- **Clicking check boxes.** You may be asked to click check boxes instead of an oval when more than one choice within a set of answers can be selected.
- **Clicking parts of a graphic.** In some questions, you will select your answers by clicking on a location (or locations) on a graphic such as a map or chart, as opposed to choosing your answer from a list.
- **Clicking on sentences.** In questions with reading passages, you may be asked to choose your answers by clicking on a sentence (or sentences) within the reading passage.
- **Dragging and dropping answer choices into targets on the screen.** You may be asked to select answers from a list of choices and drag your answers to the appropriate location in a table, paragraph of text or graphic.
- **Selecting answer choices from a drop-down menu.** You may be asked to choose answers by selecting choices from a drop-down menu (e.g., to complete a sentence).

Remember that with every question you will get clear instructions.

Perhaps the best way to understand computer-delivered questions is to view the Computer-delivered Testing Demonstration on the Praxis web site to learn how a computer-delivered test works and see examples of some types of questions you may encounter.

Understanding Selected-Response Questions

Many selected-response questions begin with the phrase "which of the following." Take a look at this example:

Which of the following is a flavor made from beans?

(A) Strawberry

(B) Cherry

(C) Vanilla

(D) Mint

How would you answer this question?

All of the answer choices are flavors. Your job is to decide which of the flavors is the one made from beans.

Try following these steps to select the correct answer.

1) **Limit your answer to the choices given.** You may know that chocolate and coffee are also flavors made from beans, but they are not listed. Rather than thinking of other possible answers, focus only on the choices given ("which of the following").

2) **Eliminate incorrect answers.** You may know that strawberry and cherry flavors are made from fruit and that mint flavor is made from a plant. That leaves vanilla as the only possible answer.

3) **Verify your answer.** You can substitute "vanilla" for the phrase "which of the following" and turn the question into this statement: "Vanilla is a flavor made from beans." This will help you be sure that your answer is correct. If you're still uncertain, try substituting the other choices to see if they make sense. You may want to use this technique as you answer selected-response questions on the practice tests.

Try a more challenging example

The vanilla bean question is pretty straightforward, but you'll find that more challenging questions have a similar structure. For example:

Entries in outlines are generally arranged according to which of the following relationships of ideas?

(A) Literal and inferential

(B) Concrete and abstract

(C) Linear and recursive

(D) Main and subordinate

You'll notice that this example also contains the phrase "which of the following." This phrase helps you determine that your answer will be a "relationship of ideas" from the choices provided. You are supposed to find the choice that describes how entries, or ideas, in outlines are related.

Sometimes it helps to put the question in your own words. Here, you could paraphrase the question in this way: "How are outlines usually organized?" Since the ideas in outlines usually appear as main ideas and subordinate ideas, the answer is (D).

QUICK TIP: Don't be intimidated by words you may not understand. It might be easy to be thrown by words like "recursive" or "inferential." Read carefully to understand the question and look for an answer that fits. An outline is something you are probably familiar with and expect to teach to your students. So slow down, and use what you know.

Watch out for selected-response questions containing "NOT," "LEAST," and "EXCEPT"

This type of question asks you to select the choice that does not fit. You must be very careful because it is easy to forget that you are selecting the negative. This question type is used in situations in which there are several good solutions or ways to approach something, but also a clearly wrong way.

How to approach questions about graphs, tables, or reading passages

When answering questions about graphs, tables, or reading passages, provide only the information that the questions ask for. In the case of a map or graph, you might want to read the questions first, and then look at the map or graph. In the case of a long reading passage, you might want to go ahead and read the passage first, noting places you think are important, and then answer the questions. Again, the important thing is to be sure you answer the questions as they refer to the material presented. So read the questions carefully.

How to approach unfamiliar formats

New question formats are developed from time to time to find new ways of assessing knowledge. Tests may include audio and video components, such as a movie clip or animation, instead of a map or reading passage. Other tests may allow you to zoom in on details in a graphic or picture.

Tests may also include interactive questions. These questions take advantage of technology to assess knowledge and skills in ways that standard selected-response questions cannot. If you see a format you are not familiar with, **read the directions carefully**. The directions always give clear instructions on how you are expected to respond.

QUICK TIP: Don't make the questions more difficult than they are. Don't read for hidden meanings or tricks. There are no trick questions on *Praxis* tests. They are intended to be serious, straightforward tests of your knowledge.

Understanding Constructed-Response Questions

Constructed-response questions require you to demonstrate your knowledge in a subject area by creating your own response to particular topics. Essays and short-answer questions are types of constructed-response questions.

For example, an essay question might present you with a topic and ask you to discuss the extent to which you agree or disagree with the opinion stated. You must support your position with specific reasons and examples from your own experience, observations, or reading.

Take a look at a few sample essay topics:

- "Celebrities have a tremendous influence on the young, and for that reason, they have a responsibility to act as role models."
- "We are constantly bombarded by advertisements—on television and radio, in newspapers and magazines, on highway signs, and the sides of buses. They have become too pervasive. It's time to put limits on advertising."
- "Advances in computer technology have made the classroom unnecessary, since students and teachers are able to communicate with one another from computer terminals at home or at work."

Keep these things in mind when you respond to a constructed-response question

1) **Answer the question accurately.** Analyze what each part of the question is asking you to do. If the question asks you to describe or discuss, you should provide more than just a list.

2) **Answer the question completely.** If a question asks you to do three distinct things in your response, you should cover all three things for the best score. Otherwise, no matter how well you write, you will not be awarded full credit.

3) **Answer the question that is asked.** Do not change the question or challenge the basis of the question. You will receive no credit or a low score if you answer another question or if you state, for example, that there is no possible answer.

4) **Give a thorough and detailed response.** You must demonstrate that you have a thorough understanding of the subject matter. However, your response should be straightforward and not filled with unnecessary information.

5) **Reread your response.** Check that you have written what you thought you wrote. Be sure not to leave sentences unfinished or omit clarifying information.

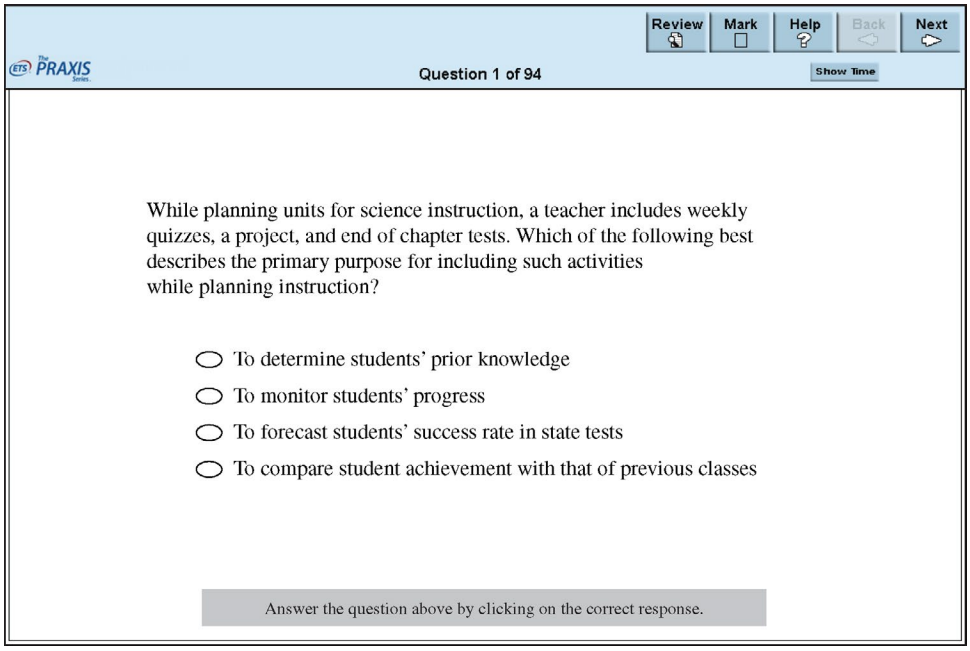
QUICK TIP: You may find that it helps to take notes on scratch paper so that you don't miss any details. Then you'll be sure to have all the information you need to answer the question.

3. Practice with Sample Test Questions

Answer practice questions and find explanations for correct answers

Sample Test Questions

This test is available via computer delivery. To illustrate what the computer-delivered test looks like, the following sample question shows an actual screen used in a computer-delivered test. For the purposes of this guide, sample questions are provided as they would appear in a paper-delivered test.



Selected Notations, Definitions, and Formulas (as provided with the test)

NOTATIONS

(a, b)	$\{x : a < x < b\}$
$[a, b)$	$\{x : a \le x < b\}$
$(a, b]$	$\{x : a < x \le b\}$
$[a, b]$	$\{x : a \le x \le b\}$
$\gcd(m, n)$	**greatest common divisor** of two integers m and n
$\text{lcm}(m, n)$	**least common multiple** of two integers m and n
$[x]$	**greatest integer** m such that $m \le x$
$m \equiv k \pmod{n}$	m and k are **congruent modulo *n*** (m and k have the same remainder when divided by n, or equivalently, $m - k$ is a multiple of n)
f^{-1}	**inverse** of an invertible function f; (**not** to be read as $\frac{1}{f}$)
$\lim_{x \to a^+} f(x)$	**right-hand limit** of $f(x)$; limit of $f(x)$ as x approaches a from the right
$\lim_{x \to a^-} f(x)$	**left-hand limit** of $f(x)$; limit of $f(x)$ as x approaches a from the left
$\varnothing$	the empty set
$x \in S$	x is an element of set S
$S \subset T$	set S is a proper subset of set T
$S \subseteq T$	either set S is a proper subset of set T or $S = T$
$\overline{S}$	complement of set S; the set of all elements not in S that are in some specified universal set
$T \setminus S$	relative complement of set S in set T, i.e., the set of all elements of T that are not elements of S
$S \cup T$	union of sets S and T
$S \cap T$	intersection of sets S and T

DEFINITIONS

A relation $\Re$ on a set S

reflexive if $x \Re x$ for all $x \in S$

symmetric if $x \Re y \Rightarrow y \Re x$ for all $x, y \in S$

transitive if $(x \Re y \text{ and } y \Re z) \Rightarrow x \Re z$ for all $x, y, z \in S$

antisymmetric if $(x \Re y \text{ and } y \Re x) \Rightarrow x = y$ for all $x, y \in S$

An equivalence relation is a reflexive, symmetric, and transitive relation.

FORMULAS

Sum

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$$

Half-angle (sign depends on the quadrant of $\frac{\theta}{2}$)

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$$

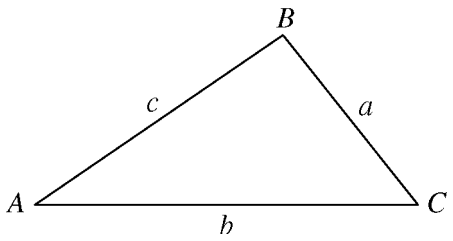
$$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$$

Range of Inverse Trigonometric Functions

$\sin^{-1} x$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$\cos^{-1} x$	$[0, \pi]$
$\tan^{-1} x$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Law of Sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$



Law of Cosines

$$c^2 = a^2 + b^2 - 2ab(\cos C)$$

DeMoivre's Theorem

$$(\cos\theta + i\sin\theta)^k = \cos(k\theta) + i\sin(k\theta)$$

Coordinate Transformation

Rectangular (x, y) to polar (r, θ) : $r^2 = x^2 + y^2$; $\tan\theta = \frac{y}{x}$ if $x \neq 0$

Polar (r, θ) to rectangular (x, y) : $x = r\cos\theta$; $y = r\sin\theta$

Distance from point (x_1, y_1) to line $Ax + By + C = 0$

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

Volume

Sphere with radius r: $V = \frac{4}{3}\pi r^3$

Right circular cone with height h and base of radius r: $V = \frac{1}{3}\pi r^2 h$

Right circular cylinder with height h and base of radius r: $V = \pi r^2 h$

Pyramid with height h and base of area B: $V = \frac{1}{3}Bh$

Right prism with height h and base of area B: $V = Bh$

Surface Area

Sphere with radius r: $A = 4\pi r^2$

Right circular cone with radius r and slant height s: $A = \pi rs + \pi r^2$

Differentiation

$$\big(f(x)g(x)\big)' = f'(x)g(x) + f(x)g'(x)$$

$$\big(f(g(x))\big)' = f'(g(x))g'(x)$$

$$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2} \text{ if } g(x) \neq 0$$

Integration by Parts

$$\int u\,dv = uv - \int v\,du$$

The sample questions that follow illustrate the kinds of questions on the test. They are not, however, representative of the entire scope of the test in either content or difficulty. Answers with explanations follow the questions.

Directions: In this chapter, questions 1–8, 11–12, and 14–18 are followed by four suggested answers or completions. Select the one that is best in each case. For innovative question types, enter your answer in the appropriate answer box (questions 9–10) or select one or more answers (question 13).

Number and Quantity

1. If *x* and *y* are even numbers and $z = 2x^2 + 4y^2$, then the greatest even number that must be a divisor of *z* is

   (A) 2
   (B) 4
   (C) 8
   (D) 16

2. For what value of *x* is the matrix $\begin{pmatrix} 1 & 4 \\ x & 6 \end{pmatrix}$ NOT invertible?

   (A) $-\frac{3}{2}$
   (B) 0
   (C) $\frac{3}{2}$
   (D) 2

Algebra

3. Jerry is 50 inches tall and is growing at the rate of $\frac{1}{24}$ inch per month. Adam is 47 inches tall and is growing at the rate of $\frac{1}{8}$ inch per month. If they each continue to grow at these rates for the next four years, after how many months will they be the same height?

   (A) 24
   (B) 30
   (C) 36
   (D) 42

4. What is the units digit of 33^{408} ?

   (A) 1
   (B) 3
   (C) 7
   (D) 9

5. For which of the following values of *k* does the equation $x^4 - 4x^2 + x + k = 0$ have four distinct real roots?

   I. –2
   II. 1
   III. 3

   (A) II only
   (B) III only
   (C) II and III only
   (D) I, II, and III

Functions

6. If $y = 5\sin x - 6$, what is the maximum value of *y* ?

   (A) -6

   (B) -1

   (C) 1

   (D) 5

$$P(t) = 250 \cdot (3.04)^{\frac{t}{1.98}}$$

7. At the beginning of 2010, the population of rabbits in a wooded area was 250. The function above was used to model the approximate population, *P*, of rabbits in the area *t* years after January 1, 2010. According to this model, which of the following best describes how the rabbit population changed in the area?

   (A) The rabbit population doubled every 4 months.

   (B) The rabbit population tripled every 6 months.

   (C) The rabbit population doubled every 36 months.

   (D) The rabbit population tripled every 24 months.

8. If $f(x) = 3x^2$, what are all real values of *a* and *b* for which the graph of $g(x) = ax^2 + b$ is below the graph of $f(x)$ for all values of *x* ?

   (A) $a \geq 3$ and *b* is positive.

   (B) $a \leq 3$ and *b* is negative.

   (C) *a* is negative and *b* is positive.

   (D) *a* is any real number and *b* is negative.

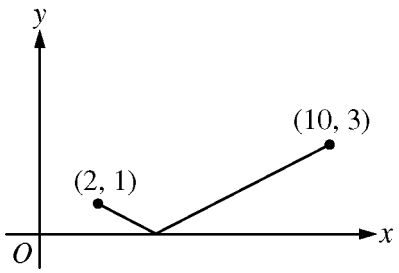
For the following question, enter your answer in the answer box.

9. If $3^{\log_3 2} + 5^{\log_5 9} = 8^{\log_8 x}$, what is the value of *x* ?

$x =$ ☐

For the following question, enter your answer in the answer boxes.

10.



The graph of the function $f(x) = a|x - b|$ on the closed interval $[2, 10]$ is shown in the *xy*-plane above. What is the value of *a* ?

Give your answer as a fraction.

$a =$ ☐/☐

Calculus

11. If $\lim_{x \to c} f(x) = 0$ and $\lim_{x \to c} g(x) = 0$, what can be concluded about the value of $\lim_{x \to c} \frac{f(x)}{g(x)}$?

    (A) The value is not finite.

    (B) The value is 0.

    (C) The value is 1.

    (D) The value cannot be determined from the information given.

12. In a certain chemical reaction, the number of grams, *N*, of a substance produced *t* hours after the reaction begins is given by $N(t) = 16t - 4t^2$, where $0 < t < 2$. At what instantaneous rate, in grams per hour, is the substance being produced 30 minutes after the reaction begins?

    (A) 7

    (B) 12

    (C) 16

    (D) 20

13. **For the following question, select all the answer choices that apply.**

    The function *f* is defined by $f(x) = \int_0^x \frac{1}{t^3 + 27} dt$ for all *x* in the interval $(-2, 2)$. Which of the following statements are true?

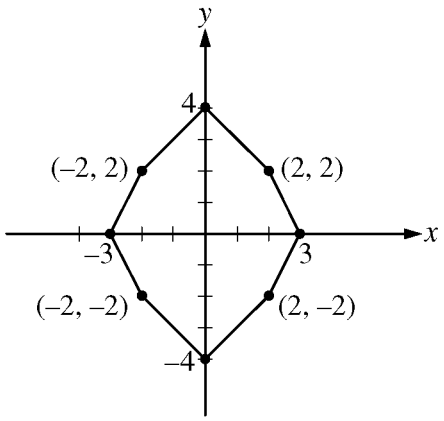
    Select all that apply.

    (A) $f(-1) < 0$

    (B) $f(0) = 0$

    (C) $f'(1) < 0$

Geometry



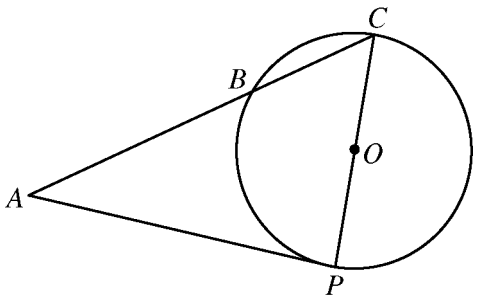
14. For how many angles θ, where $0 < \theta \leq 2\pi$, will rotation about the origin by angle θ map the octagon in the figure above onto itself?

    (A) One

    (B) Two

    (C) Four

    (D) Eight



Note: Figure not drawn to scale

15. In the circle above with center O and radius 2, tangent $\overline{AP}$ has length 3 and is tangent to the circle at *P*. If $\overline{CP}$ is a diameter of the circle, what is the length of $\overline{BC}$?

    (A) 1.25

    (B) 2

    (C) 3.2

    (D) 5

Probability and Statistics

16. The measures of the handspans of ninth-grade students at Tyler High School are approximately normally distributed, with a mean of 7 inches and a standard deviation of 1 inch. Of the following, which group is expected to have the greatest percent of measures?

    (A) The group of handspan measures that are less than 6 inches

    (B) The group of handspan measures that are greater than 7 inches

    (C) The group of handspan measures that are between 6 and 8 inches

    (D) The group of handspan measures that are between 5 and 7 inches

17. A two-sided coin is unfairly weighted so that when it is tossed, the probability that heads will result is twice the probability that tails will result. If the coin is to be tossed 3 separate times, what is the probability that tails will result on exactly 2 of the tosses?

    (A) $\frac{2}{9}$

    (B) $\frac{3}{8}$

    (C) $\frac{4}{9}$

    (D) $\frac{2}{3}$

Discrete Mathematics

18. Given the recursive function defined by $f(1) = -3$ and $f(n) = f(n-1) - 6$ for all integers $n \geq 2$, what is the value of $f(4)$?

    (A) -2

    (B) -9

    (C) -10

    (D) -21

Answers to Sample Questions

Number and Quantity

1. The correct answer is (C). Since 2 is a divisor of both $2x^2$ and $4y^2$, it follows that 2 is a divisor of *z*. To find out if there is a greater even number that must be a divisor of *z*, consider the additional information given, which is that *x* and *y* are both even numbers. Since *x* and *y* are even numbers, they can be expressed as $x = 2m$ and $y = 2n$, respectively, where *m* and *n* can be either odd or even integers. Substituting these values for *x* and *y* in the expression for *z* yields $z = 2(2m)^2 + 4(2n)^2$. It follows then that $z = 8m^2 + 16n^2$ and that 8 is a divisor of *z*. The number 16 would also be a divisor of *z* if *m* is even, but not if *m* is odd. Since *m* and *n* can be either even or odd and the question asks for the largest even number that must be a divisor of *z*, the correct answer is (C), 8.

2. The correct answer is (C). A matrix is not invertible if the determinant of the matrix is equal to zero. The determinant of the matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is equal to $ad - bc$. For the matrix given in the question, the determinant is equal to $(1)(6) - (4)(x)$. This equals 0 when $6 - 4x = 0$, or $x = \frac{3}{2}$, and the correct answer is (C).

Algebra

3. The correct answer is (C). The heights in this question can be expressed as two linear equations. Jerry's height in inches, *J*, can be expressed as $J = 50 + \frac{1}{24}m$, where *m* is the number of months from now. Adam's height in inches, *A*, can be expressed as $A = 47 + \frac{1}{8}m$. The question asks, "after how many months will they be the same height?" This is the same as asking, "for what value of *m* will *J* = *A*?" The solution can be found by solving $50 + \frac{1}{24}m = 47 + \frac{1}{8}m$ for m, as shown below.

$$50 + \frac{1}{24}m = 47 + \frac{1}{8}m$$
$$50 - 47 = \left(\frac{1}{8} - \frac{1}{24}\right)m$$
$$3 = \left(\frac{3}{24} - \frac{1}{24}\right)m$$
$$3 = \frac{1}{12}m$$
$$m = 36$$

So the correct answer is (C), 36 months.

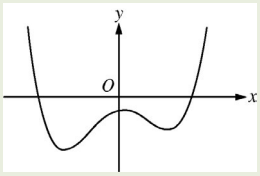
4. The correct answer is (A). To find the units digit of 33^{408}, it is helpful to find the first few integer powers of 33 and look for a pattern. For example,

$$33^1 = 33$$
$$33^2 = 1{,}089$$
$$33^3 = 35{,}937$$
$$33^4 = 1{,}185{,}921$$
$$33^5 = 39{,}135{,}393$$
$$33^6 = 1{,}291{,}467{,}969$$

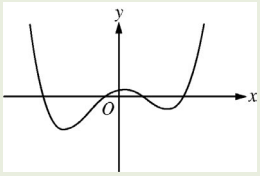
The pattern in the units digits is 3, 9, 7, 1, 3, 9, … and it will continue to repeat with every four integers of the exponent. Dividing 408 by 4 yields 102 with no remainder. Therefore, the units digit of 33^{408} will be the same as the units digit of 33^4, which is 1. So, the correct answer is (A).

5. The correct answer is (A). The fourth-degree polynomial $x^4 - 4x^2 + x + k$ has at most four distinct real roots, and the roots are the *x*-intercepts of the graph of $y = x^4 - 4x^2 + x + k$. So, one way to determine which of the three given values of *k* result in the equation $x^4 - 4x^2 + x + k = 0$ having four distinct real roots is to graph $y = x^4 - 4x^2 + x + k$ and examine the *x*-intercepts.

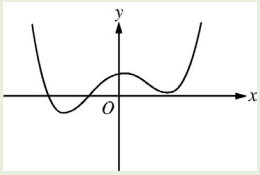
I. $x^4 - 4x^2 + x - 2 = 0$



II. $x^4 - 4x^2 + x + 1 = 0$



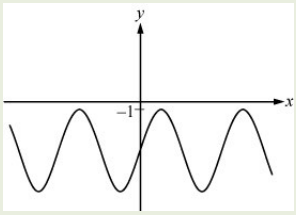
III. $x^4 - 4x^2 + x + 3 = 0$



It can be seen that the values of *k* given in I and III each result in the equation having only two distinct real roots. The value of *k* given in II results in the equation having four distinct real roots. The correct answer is (A), II only.

Functions

6. The correct answer is (B). There are two ways to answer this question. The first solution is based on reasoning about the function $f(x) = \sin x.$ First recall that the maximum value of sin *x* is 1 and, therefore, the maximum value of 5sin *x* is 5. The maximum value of $y = 5\sin x - 6$ is then $5 - 6 = -1.$ Alternatively, graph the function $y = 5\sin x - 6$ and find the maximum value of *y* from the graph.



The maximum value is –1, and the correct answer is (B).

7. The correct answer is (D). The question asks for a verbal description of the change in rabbit population, based on the function given. Recall the meaning of the base (growth factor) and the exponent in an exponential growth model. Note that the function given $P(t) = 250 \cdot (3.04)^{\frac{t}{1.98}} \approx 250 \cdot 3^{\frac{t}{2}}.$ Observe from this approximation, with base 3 and exponent $\frac{t}{2}$, that the population tripled every two years. In fact, $(3.04)^{\frac{2}{1.98}} \approx 3.07$, so the population tripled in a time period of a little less than 2 years. Thus, the correct answer is (D), "The rabbit population tripled every 24 months."

8. The correct answer is (B). Since the graph of function *g* is below the graph of function *f* for all values of *x*, then $ax^2 + b < 3x^2$ for all values of *x*; that is, $(a-3)x^2 + b < 0$ for all values of *x*. In particular, substituting 0 for *x* in the last inequality gives $(a-3)(0)+b<0$, or $b<0$, so *b* is negative. If *a* were greater than 3, then the graph of $y=(a-3)x^2+b$ would be a parabola that opens upward and there would be values of *x* that would make $(a-3)x^2+b$ positive, which contradicts the information that $(a-3)x^2+b<0$ for all values of *x*; this contradiction means that $a \le 3$ must be true. Therefore, the correct answer is (B).

9. The correct answer is 11. Recall that logarithmic functions are the inverse functions of exponential functions and that, more specifically, we have the identity $a^{\log_a b} = b$ for any positive numbers *a* and *b*, where $a \neq 1$. Since $3^{\log_3 2} = 2,\ 5^{\log_5 9} = 9,$ and $8^{\log_8 x} = x,$ the equation in the problem is equivalent to $2+9=x.$ Therefore, the value of *x* is 11.

10. The correct answer is $\frac{1}{2}$. The graph of the function *f* consists of two line segments that have a common end point at the point $(b, 0)$ on the *x*-axis. Since the graph of $y=f(x)$ lies on or above the *x*-axis, *a* is positive. The slope of the left line segment [with end points at $(2,1)$ and $(b,0)$] is $-a$, and the slope of the right line segment [with end points at $(b,0)$ and $(10,3)$] is *a*. Therefore,

$$a = -\frac{0-1}{b-2} = \frac{3-0}{10-b}.$$

Solving for *b* gives $b=4,$ which implies that $a=\frac{1}{2}.$

Calculus

11. The correct answer is (D). Although both f and g have the limit 0 as $x \to 0,$ one function might be approaching 0 more quickly than the other, which would affect the value of the limit of the quotient. For example, if one of the functions is x and the other is $x^2,$ then the quotient is either x or $\frac{1}{x},$ and so the limit of the quotient is either 0 or nonexistent, respectively. In fact, the value of the limit can be any nonzero real number *b*, as shown by $\lim_{x\to 0}\frac{bx}{x} = b$. Thus, answer choices (A), (B), and (C) are incorrect and the correct answer is (D).

12. The correct answer is (B). The instantaneous rate of change in the number of grams of substance produced 30 minutes after the reaction begins is the value of the first derivative of *N* evaluated at 30 minutes. First, convert 30 minutes into hours, then evaluate the first derivative of *N* at that value of *t*. Since 30 minutes equals $\frac{1}{2}$ hour, you will need to evaluate $N'\left(\frac{1}{2}\right).$

First, find $N'(t)$.

$$N'(t) = 16 - 8t.$$

Therefore, $N'\left(\frac{1}{2}\right) = 16 - 8\left(\frac{1}{2}\right) = 12.$ The answer is 12 grams per hour, so the correct answer is (B).

13. The correct answers are (A) and (B).

Choice (A): Recall that swapping the end points of a definite integral changes the sign of the integral. Thus,

$$f(-1) = \int_0^{-1} \frac{1}{t^3+27}\,dt = -\int_{-1}^{0} \frac{1}{t^3+27}\,dt.$$

Since $\frac{1}{t^3+27}$ is a continuous function of *t* on the closed interval $[-2, 2]$, and since $\frac{1}{t^3+27} > 0$ for $t \in [-2, 2]$, then $\int_{-1}^{0} \frac{1}{t^3+27}\,dt > 0,$ and, therefore, $f(-1) = -\int_{-1}^{0} \frac{1}{t^3+27}\,dt < 0.$ The statement in choice (A) is true.

Choice (B): Recall that the value of a definite integral with the same value for the two end points is zero. Thus, $f(0)=\int_0^0 \frac{1}{t^3+27}\,dt=0$. The statement in choice (B) is true.

Choice (C): Recall that the fundamental theorem of calculus applied to function *f* implies that

$f'(x)=\frac{1}{x^3+27}$. In particular, $f'(1)=\frac{1}{1^3+27}=\frac{1}{28}>0$.

The statement in choice (C) is false. Therefore, the correct answers are (A) and (B).

Geometry

14. The correct answer is (B). To begin, consider a single point on the octagon, such as the point $(0, 4)$ at the top of the octagon in the figure. This point is 4 units from the origin, so any rotation that maps the octagon onto itself would need to map this point onto a point that is also 4 units from the origin. The only other point on the octagon that is 4 units from the origin is the point $(0, -4)$. A rotation of angle $\theta = \pi$ would map the point $(0, 4)$ onto the point $(0, -4)$. The octagon is symmetric about the *x*- and *y*-axes, so a rotation of angle $\theta = \pi$ would map all of the points of the octagon onto corresponding points of the octagon. Likewise, a rotation of angle $\theta = 2\pi$ would map the point $(0, 4)$ onto itself (and map all other points of the octagon onto themselves). No other values of θ such that $0 < \theta \leq 2\pi$ would map the octagon onto itself. Therefore, the correct answer is two, choice (B).

15. The correct answer is (C). To determine the length of $\overline{BC}$, it would be helpful to first label the figure with the information given. Since the circle has radius 2, then both $\overline{OC}$ and $\overline{OP}$ have length 2 and $\overline{CP}$ has length 4. $\overline{AP}$ is tangent to the circle at *P*, so angle *APC* is a right angle. The length of $\overline{AP}$ is given as 3. This means that triangle *ACP* is a 3-4-5 right triangle and $\overline{AC}$ has length 5. Notice that since $\overline{CP}$ is a diameter of the circle, angle *CBP* is also a right angle. Angle *BCP* is in both triangle *ACP* and triangle *PCB,* and, therefore, the two triangles are similar. Then find the length of $\overline{BC}$ by setting up a proportion between the corresponding parts of the similar triangles as follows:

$$\frac{CP}{AC}=\frac{BC}{PC}$$

$$\frac{4}{5}=\frac{BC}{4}$$

$$BC=\frac{16}{5}=3.2$$

The correct answer is (C), 3.2.

Probability and Statistics

16. The correct answer is (C). Recall that approximately 68% of a normally distributed set of data lie within ± 1 standard deviation of the mean and approximately 95% of the data lie within ± 2 standard deviations of the mean. Evaluate each answer choice in order to determine which of the groups has the greatest percent.

Choice (A): Since the mean handspan is 7 inches and the standard deviation is 1 inch, the group of handspan measures that is less than 6 inches is the group that is more than 1 standard deviation less than the mean. The group of handspan measures that is less than 7 inches includes 50% of the measures. Approximately 34 percent ($\frac{1}{2}$ of 68 percent) of the measures are between 6 inches and 7 inches (within 1 standard deviation less than the mean). So, the group with handspan measures less than 6 inches would be approximately equal to $\mathbf{50\% - 34\%}$, or 16% of the measures.

Choice (B): Since 7 inches is the mean, approximately 50% of the measures are greater than the mean.

Choice (C): This group is within ± 1 standard deviation of the mean. This group contains approximately 68% of the measures.

Choice (D): This group is between the mean and 2 standard deviations less than the mean. Approximately 47.5% ($\frac{1}{2}$ of 95%) of the measures are between 5 inches and 7 inches.

Of the answer choices given, the group described in (C) is expected to contain the greatest percent of the measures, approximately 68%, so (C) is the correct answer.

17. The correct answer is (A). Because each toss of the coin is an independent event, the probability of tossing heads then 2 tails, $P(HTT)$, is equal to $P(H)\cdot P(T)\cdot P(T)$, where $P(H)$ is the probability of tossing heads and $P(T)$ is the probability of tossing tails. The probability of tossing heads is twice the probability of tossing tails, so $P(H)=\frac{2}{3}$ and $P(T)=\frac{1}{3}$.

Therefore, $P(HTT)=\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)\left(\frac{1}{3}\right)=\frac{2}{27}$. There are 3 ways in which exactly 2 of 3 tosses would be tails, and each of them has an equal probability of occurring: $P(THT)=P(TTH)=P(HTT)=\frac{2}{27}$. Therefore, the total probability that tails will result exactly 2 times in 3 tosses is $3\left(\frac{2}{27}\right)=\frac{2}{9}$. So, the correct answer is (A).

Discrete Mathematics

18. The correct answer is (D). Using the recursive definition and the initial condition, the value of $f(4)$, can be computed as follows.

$$f(1)=-3$$
$$f(2)=f(1)-6=-3-6=-9$$
$$f(3)=f(2)-6=-9-6=-15$$
$$f(4)=f(3)-6=-15-6=-21$$

The correct answer is (D).

4. Determine Your Strategy for Success

Set clear goals and deadlines so your test preparation is focused and efficient

Effective *Praxis* test preparation doesn't just happen. You'll want to set clear goals and deadlines for yourself along the way. Otherwise, you may not feel ready and confident on test day.

1) Learn what the test covers.

You may have heard that there are several different versions of the same test. It's true. You may take one version of the test and your friend may take a different version a few months later. Each test has different questions covering the same subject area, but both versions of the test measure the same skills and content knowledge.

You'll find specific information on the test you're taking on page 5, which outlines the content categories that the test measures and what percentage of the test covers each topic. Visit www.ets.org/praxis/testprep for information on other *Praxis* tests.

2) Assess how well you know the content.

Research shows that test takers tend to overestimate their preparedness—this is why some test takers assume they did well and then find out they did not pass.

The *Praxis* tests are demanding enough to require serious review of likely content, and the longer you've been away from the content, the more preparation you will most likely need. If it has been longer than a few months since you've studied your content area, make a concerted effort to prepare.

3) Collect study materials.

Gathering and organizing your materials for review are critical steps in preparing for the *Praxis* tests. Consider the following reference sources as you plan your study:

- Did you take a course in which the content area was covered? If yes, do you still have your books or your notes?
- Does your local library have a high school-level textbook in this area? Does your college library have a good introductory college-level textbook in this area?

Practice materials are available for purchase for many *Praxis* tests at www.ets.org/praxis/testprep. Test preparation materials include sample questions and answers with explanations.

4) Plan and organize your time.

You can begin to plan and organize your time while you are still collecting materials. Allow yourself plenty of review time to avoid cramming new material at the end. Here are a few tips:

- Choose a test date far enough in the future to leave you plenty of preparation time. Test dates can be found at www.ets.org/praxis/register/centers_dates.
- Work backward from that date to figure out how much time you will need for review.
- Set a realistic schedule—and stick to it.

5) Practice explaining the key concepts.

Praxis tests with constructed-response questions assess your ability to explain material effectively. As a teacher, you'll need to be able to explain concepts and processes to students in a clear, understandable way. What are the major concepts you will be required to teach? Can you explain them in your own words accurately, completely, and clearly? Practice explaining these concepts to test your ability to effectively explain what you know.

6) Understand how questions will be scored.

Scoring information can be found on page 63.

7) Develop a study plan.

A study plan provides a road map to prepare for the *Praxis* tests. It can help you understand what skills and knowledge are covered on the test and where to focus your attention. Use the study plan template on page 40 to organize your efforts.

And most important—get started!

Would a Study Group Work for You?

Using this guide as part of a study group

People who have a lot of studying to do sometimes find it helpful to form a study group with others who are working toward the same goal. Study groups give members opportunities to ask questions and get detailed answers. In a group, some members usually have a better understanding of certain topics, while others in the group may be better at other topics. As members take turns explaining concepts to one another, everyone builds self-confidence.

If the group encounters a question that none of the members can answer well, the group can go to a teacher or other expert and get answers efficiently. Because study groups schedule regular meetings, members study in a more disciplined fashion. They also gain emotional support. The group should be large enough so that multiple people can contribute different kinds of knowledge, but small enough so that it stays focused. Often, three to six members is a good size.

Here are some ways to use this guide as part of a study group:

- **Plan the group's study program.** Parts of the study plan template, beginning on page 43, can help to structure your group's study program. By filling out the first five columns and sharing the worksheets, everyone will learn more about your group's mix of abilities and about the resources, such as textbooks, that members can share with the group. In the sixth column ("Dates I will study the content"), you can create an overall schedule for your group's study program.
- **Plan individual group sessions.** At the end of each session, the group should decide what specific topics will be covered at the next meeting and who will present each topic. Use the topic headings and subheadings in the Test at a Glance table on page 5 to select topics, and then select practice questions, beginning on page 22.
- **Prepare your presentation for the group.** When it's your turn to present, prepare something that is more than a lecture. Write two or three original questions to pose to the group. Practicing writing actual questions can help you better understand the topics covered on the test as well as the types of questions you will encounter on the test. It will also give other members of the group extra practice at answering questions.

- **Take a practice test together.** The idea of a practice test is to simulate an actual administration of the test, so scheduling a test session with the group will add to the realism and may also help boost everyone's confidence. Remember, complete the practice test using only the time that will be allotted for that test on your administration day.
- **Learn from the results of the practice test.** Review the results of the practice test, including the number of questions answered correctly in each content category. For tests that contain constructed-response questions, look at the Sample Test Questions section, which also contain sample responses to those questions and shows how they were scored. Then try to follow the same guidelines that the test scorers use.
- **Be as critical as you can.** You're not doing your study partner(s) any favors by letting them get away with an answer that does not cover all parts of the question adequately.
- **Be specific.** Write comments that are as detailed as the comments about the sample responses. Indicate where and how your study partner(s) are doing an inadequate job of answering the question. Writing notes in the margins of the answer sheet may also help.
- **Be supportive.** Include comments that point out what your study partner(s) got right.

Then plan one or more study sessions based on aspects of the questions on which group members performed poorly. For example, each group member might be responsible for rewriting one paragraph of a response in which someone else did an inadequate job.

Whether you decide to study alone or with a group, remember that the best way to prepare is to have an organized plan. The plan should set goals based on specific topics and skills that you need to learn, and it should commit you to a realistic set of deadlines for meeting those goals. Then you need to discipline yourself to stick with your plan and accomplish your goals on schedule.

5. Develop Your Study Plan

Develop a personalized study plan and schedule

Planning your study time is important because it will help ensure that you review all content areas covered on the test. Use the sample study plan below as a guide. It shows a plan for the *Core Academic Skills for Educators: Reading* test. Following that is a study plan template that you can fill out to create your own plan. Use the "Learn about Your Test" and "Topics Covered" information beginning on page 5 to help complete it.

Use this worksheet to:
1. **Define Content Areas:** List the most important content areas for your test as defined in chapter 1.
2. **Determine Strengths and Weaknesses:** Identify your strengths and weaknesses in each content area.
3. **Identify Resources:** Identify the books, courses, and other resources you plan to use for each content area.
4. **Study:** Create and commit to a schedule that provides for regular study periods.

Praxis Test Name (Test Code): Core Academic Skills for Educators: Reading (5712)
Test Date: 9/15/15

Content covered	Description of content	How well do I know the content? (scale 1–5)	What resources do I have/need for the content?	Where can I find the resources I need?	Dates I will study the content	Date completed
Key Ideas and Details						
Close reading	Draw inferences and implications from the directly stated content of a reading selection	3	Middle school English textbook	College library, middle school teacher	7/15/15	7/15/15
Determining Ideas	Identify summaries or paraphrases of the main idea or primary purpose of a reading selection	3	Middle school English textbook	College library, middle school teacher	7/17/15	7/17/15
Determining Ideas	Identify summaries or paraphrases of the supporting ideas and specific details in a reading selection	3	Middle and high school English textbook	College library, middle and high school teachers	7/20/15	7/21/15
Craft, Structure, and Language Skills						
Interpreting tone	Determine the author's attitude toward material discussed in a reading selection	4	Middle and high school English textbook	College library, middle and high school teachers	7/25/15	7/26/15
Analysis of structure	Identify key transition words and phrases in a reading selection and how they are used	3	Middle and high school English textbook, dictionary	College library, middle and high school teachers	7/25/15	7/27/15
Analysis of structure	Identify how a reading selection is organized in terms of cause/effect, compare/contrast, problem/solution, etc.	5	High school textbook, college course notes	College library, course notes, high school teacher, college professor	8/1/15	8/1/15
Author's purpose	Determine the role that an idea, reference, or piece of information plays in an author's discussion or argument	5	High school textbook, college course notes	College library, course notes, high school teacher, college professor	8/1/15	8/1/15

(continued on next page)

Content covered	Description of content	How well do I know the content? (scale 1–5)	What resources do I have/need for the content?	Where can I find the resources I need?	Dates I will study the content	Date completed
Language in different contexts	Determine whether information presented in a reading selection is presented as fact or opinion	4	High school textbook, college course notes	College library, course notes, high school teacher, college professor	8/1/15	8/1/15
Contextual meaning	Identify the meanings of words as they are used in the context of a reading selection	2	High school textbook, college course notes	College library, course notes, high school teacher, college professor	8/1/15	8/1/15
Figurative Language	Understand figurative language and nuances in word meanings	2	High school textbook, college course notes	College library, course notes, high school teacher, college professor	8/8/15	8/8/15
Vocabulary range	Understand a range of words and phrases sufficient for reading at the college and career readiness level	2	High school textbook, college course notes	College library, course notes, high school teacher, college professor	8/15/15	8/17/15
Integration of Knowledge and Ideas						
Diverse media and formats	Analyze content presented in diverse media and formats, including visually and quantitatively, as well as in words	2	High school textbook, college course notes	College library, course notes, high school teacher, college professor	8/22/15	8/24/15
Evaluation of arguments	Identify the relationship among ideas presented in a reading selection	4	High school textbook, college course notes	College library, course notes, high school teacher, college professor	8/24/15	8/24/15
Evaluation of arguments	Determine whether evidence strengthens, weakens, or is relevant to the arguments in a reading selection	3	High school textbook, college course notes	College library, course notes, high school teacher, college professor	8/27/15	8/27/15
Evaluation of arguments	Determine the logical assumptions upon which an argument or conclusion is based	5	High school textbook, college course notes	College library, course notes, high school teacher, college professor	8/28/15	8/30/15
Evaluation of arguments	Draw conclusions from material presented in a reading selection	5	High school textbook, college course notes	College library, course notes, high school teacher, college professor	8/30/15	8/31/15
Comparison of texts	Recognize or predict ideas or situations that are extensions of or similar to what has been presented in a reading selection	4	High school textbook, college course notes	College library, course notes, high school teacher, college professor	9/3/15	9/4/15
Comparison of texts	Apply ideas presented in a reading selection to other situations	2	High school textbook, college course notes	College library, course notes, high school teacher, college professor	9/5/15	9/6/15

My Study Plan

Use this worksheet to:

1. **Define Content Areas:** List the most important content areas for your test as defined in chapter 1.
2. **Determine Strengths and Weaknesses:** Identify your strengths and weaknesses in each content area.
3. **Identify Resources:** Identify the books, courses, and other resources you plan to use for each content area.
4. **Study:** Create and commit to a schedule that provides for regular study periods.

Praxis Test Name (Test Code): _____

Test Date: _____

Content covered	Description of content	How well do I know the content? (scale 1–5)	What resources do I have/need for this content?	Where can I find the resources I need?	Dates I will study this content	Date completed

(continued on next page)

Content covered	Description of content	How well do I know the content? (scale 1–5)	What resources do I have/need for the content?	Where can I find the resources I need?	Dates I will study the content	Date completed

6. Study Topics

Detailed study topics with questions for discussion

Using the Study Topics That Follow

The Mathematics: Content Knowledge test is designed to measure the knowledge and skills necessary for a beginning teacher.

This chapter is intended to help you organize your preparation for the test and to give you a clear indication of the depth and breadth of the knowledge required for success on the test.

Virtually all accredited programs address the topics covered by the test; however, you are not expected to be an expert on all aspects of the topics that follow.

You are likely to find that the topics below are covered by most introductory textbooks. Consult materials and resources, including lecture and laboratory notes, from all your coursework. You should be able to match up specific topics and subtopics with what you have covered in your courses.

Try not to be overwhelmed by the volume and scope of content knowledge in this guide. Although a specific term may not seem familiar as you see it here, you might find you can understand it when applied to a real-life situation. Many of the items on the actual test will provide you with a context to apply to these topics or terms.

Discussion Areas

Interspersed throughout the study topics are discussion areas, presented as open-ended questions or statements. These discussion areas are intended to help test your knowledge of fundamental concepts and your ability to apply those concepts to situations in the classroom or the real world. Most of the areas require you to combine several pieces of knowledge to formulate an integrated understanding and response. If you spend time on these areas, you will gain increased understanding and facility with the subject matter covered on the test. You may want to discuss these areas and your answers with a teacher or mentor.

Note that this study companion *does **not** provide answers for the discussion area questions*, but thinking about the answers to them will help improve your understanding of fundamental concepts and will probably help you answer a broad range of questions on the test.

Study Topics

An overview of the areas covered on the test, along with their subareas, follows.

I. Number and Quantity, Algebra, Functions, and Calculus

A. Number and Quantity

1. Understand the properties of exponents
   a. perform operations involving exponents, including negative and rational exponents
   b. demonstrate an understanding of the properties of exponential expressions
   c. use the properties of exponents to rewrite expressions that have radicals or rational exponents
2. Understand the properties of rational and irrational numbers, and the interactions between those sets of numbers
   a. recognize that the sum or product of two rational numbers is rational
   b. recognize that the sum of a rational number and an irrational number is irrational
   c. recognize that the product of a nonzero rational number and an irrational number is irrational
   d. recognize that the sum or product of two irrational numbers can be rational or irrational
3. Understand how to solve problems by reasoning quantitatively (e.g., dimensional analysis, reasonableness of solutions)
   a. use units as a way to understand problems and to guide the solution of multistep problems
   b. choose and interpret units consistently in formulas
   c. choose and interpret the scale and the origin in graphs and data displays
   d. recognize the reasonableness of results within the context of a given problem
4. Understand the structure of the natural, integer, rational, real, and complex number systems and how the basic operations (+, –, ×, and ÷) on numbers in these systems are performed
   a. solve problems using addition, subtraction, multiplication, and division of rational, irrational, and complex numbers
   b. apply the order of operations
   c. given operations on a number system, determine whether the properties (e.g., commutative, associative, distributive) hold
   d. compare, classify, and order real numbers
   e. simplify and approximate radicals
   f. find conjugates of complex numbers
   g. demonstrate an understanding of the properties of counting numbers (e.g., prime, composite, prime factorization, even, odd, factors, multiples)
5. Understand how to work with complex numbers when solving polynomial equations and rewriting polynomial expressions
   a. solve quadratic equations with real coefficients that have complex solutions
   b. extend polynomial identities to the complex numbers
      (e.g., $x^2 + y^2 = (x + yi)(x - yi)$)
   c. verify the fundamental theorem of algebra for quadratic polynomials
6. Understand how to perform operations on matrices and how to use matrices in applications
   a. use matrices to represent and manipulate data
   b. multiply matrices by scalars to produce new matrices
   c. add, subtract, and multiply matrices of appropriate dimensions
   d. understand that matrix multiplication for square matrices is not a commutative operation but still satisfies the associative and distributive properties
   e. understand the role played by zero and identity matrices in matrix addition and multiplication
   f. understand that the determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse

g. work with 2×2 matrices as transformations of the plane and interpret the absolute value of the determinant in terms of area

7. Understand how to solve problems involving ratios, proportions, averages, percents, and metric and traditional unit conversions
   a. apply the concept of a ratio and use ratio language and notation to describe a relationship between two quantities
   b. compute unit rates
   c. use ratio reasoning to convert rates
   d. solve problems involving scale factors
   e. recognize and represent proportional and inversely proportional relationships between two quantities
   f. use proportional relationships to solve multistep ratio, average, and percent problems
   g. solve measurement and estimation problems involving time, length, temperature, volume, and mass in both the U.S. customary system and the metric system, where appropriate
   h. convert units within the metric and U.S. customary systems
8. Know how to analyze both precision and accuracy in measurement situations
   a. choose a level of accuracy appropriate to limitations on measurement when reporting quantities
   b. calculate or estimate absolute and relative error in the numerical answer to a problem
9. Understand various ways to represent and compare very large and very small numbers (e.g., scientific notation, orders of magnitude)
   a. represent and compare very large and very small numbers
10. Understand how to both estimate and perform calculations on very large and very small quantities
    a. use orders of magnitude to estimate very large and very small numbers
    b. perform calculations on numbers in scientific notation

Discussion areas: number and Quantity

- Can you use the properties of exponents to simplify and rearrange expressions?
- Can you identify the result of arithmetic operations on rational and irrational numbers as either rational or irrational?
- Can you simplify expressions that contain radicals or rational exponents?
- Can you verify that radical expressions are equivalent numerically and analytically?
- Can you perform arithmetic operations on real and complex numbers?
- Can you apply the order of operations in arithmetic computations?
- Can you recognize and use $a - bi$ as the conjugate of $a + bi$?
- Can you solve quadratic equations with real solutions and complex solutions?
- Can you perform arithmetic operations with matrices when possible?
- Can you find the determinant of a 2×2 or 3×3 square matrix and understand that if the determinant is zero, the matrix does not have a multiplicative inverse?
- Can you compute or identify a ratio or rate?
- Can you use proportional relationships to compute percents?
- Can you convert between units—for example, converting inches to meters?
- Can you solve problems using units to guide the solution?
- Can you solve measurement problems involving time, length, temperature, volume, and mass?
- Can you calculate measurement error given an observed measurement and a calculated measurement?
- Can you identify and represent very small and very large numbers in scientific notation?
- Can you do calculations involving scientific notation?

B. Algebra

1. Understand how to write algebraic expressions in equivalent forms
   a. use the structure of an expression to identify ways to rewrite it
   b. understand how to rewrite quadratic expressions for specific purposes (e.g., factoring/finding zeros, completing the square/finding maxima or minima)
   c. use the properties of exponents to rewrite expressions for exponential functions
2. Understand how to perform arithmetic operations on polynomials
   a. add, subtract, and multiply polynomials
3. Understand the relationship between zeros of polynomial functions (including their graphical representation) and factors of the related polynomial expressions
   a. know and apply the remainder theorem: for a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$
   b. use factorization to identify zeros of polynomials
   c. use zeros of a polynomial to construct a rough graph of the function defined by the polynomial
4. Understand how to use polynomial identities (e.g., difference of squares, sum and difference of cubes) to solve problems
   a. apply the binomial theorem for the expansion of $(x+y)^n$ in powers of x and y for a positive integer n
5. Understand how to rewrite rational expressions and perform arithmetic operations on rational expressions
   a. rewrite simple rational expressions in different forms
   b. understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression
   c. add, subtract, multiply, and divide rational expressions
6. Understand how to create equations and inequalities that describe relationships
   a. create equations and inequalities in one variable and use them to solve problems and graph solutions on the number line
   b. create equations and inequalities in two or more variables to represent relationships between quantities, solve problems, and graph them on the coordinate plane with labels and scales
   c. represent constraints by equations, inequalities, or systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context
   d. rearrange formulas to highlight a quantity of interest (e.g., solve $d = rt$ for t)
7. Understand how to justify the reasoning process used to solve equations, including analysis of potential extraneous solutions
   a. explain each step in solving a simple equation
   b. solve simple rational and radical equations in one variable, incorporating analysis of possible extraneous solutions
8. Understand how varied techniques (e.g., graphical, algebraic) are used to solve equations and inequalities in one variable
   a. solve linear equations and inequalities in one variable, including equations with coefficients represented by letters
   b. use the method of completing the square to transform any quadratic equation in x into the equivalent form $(x-p)^2 = q$.
   c. solve equations using a variety of methods (e.g., using graphs, using the quadratic formula, factoring)
   d. use different methods (e.g., discriminant analysis, graphical analysis) to determine the nature of the solutions of a quadratic equation
   e. write complex solutions in the form $a \pm bi$.

9. Understand how varied techniques (e.g., graphical, algebraic, matrix) are used to solve systems of equations and inequalities
    a. explain why, when solving a system of two equations using the elimination method, replacing one or both equations with a scalar multiple produces a system with the same solutions as the solutions of the original system
    b. solve a system consisting of two linear equations in two variables algebraically and graphically
    c. solve a system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically
    d. represent a system of linear equations as a single matrix equation
    e. find the inverse of a matrix, if it exists, and use it to solve systems of linear equations
    f. explain why the x-coordinates of the intersection points of the graphs of $y = f(x)$ and $y = g(x)$ are the solutions of $f(x) = g(x)$
    g. find the solutions of $f(x) = g(x)$ approximately (e.g., use technology to graph the functions, make tables of values, find successive approximations); include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, or logarithmic functions
    h. graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes
10. Understand the properties of number systems under various operations
    a. given operations on algebraic expressions, determine whether the properties hold (e.g., commutative, associative, distributive)
11. Understand the concept of rate of change of nonlinear functions
    a. calculate and interpret the average rate of change of a function presented symbolically, numerically, or graphically over a specified interval
12. Understand the concepts of intercept(s) of a line and slope as a rate of change
    a. calculate and interpret the intercepts of a line
    b. calculate and interpret the slope of a line presented symbolically, numerically, or graphically
    c. estimate the rate of change of a linear function from a graph
13. Understand how to find the zero(s) of functions
    a. use a variety of techniques to find and analyze the zero(s) (real and complex) of functions

Discussion areas: Algebra

- Can you rewrite quadratic expressions to find zeros, complete the square, and find the relative extrema?
- Can you add, subtract, and multiply polynomials?
- Can you use the remainder theorem and factor theorem for polynomials?
- Can you find and use zeros to sketch a graph of the function?
- Can you calculate the coefficient for a term in the expansion of $(x + y)^n$?
- Can you verify binomial expansion by evaluating expressions for values of the variables?
- Can you add, subtract, multiply, and divide rational expressions?
- Can you use linear equations or linear inequalities to model real-life problems?
- Can you solve linear equations and linear inequalities algebraically?
- Can you graph the solution of a linear inequality in one variable on the number line and the solution of a linear inequality in two variables on the coordinate plane?
- Can you solve for the variable of interest in a formula?
- Can you identify when extraneous solutions may occur in solving rational and radical equations?

- Can you solve quadratic equations by completing the square, factoring, and using the quadratic formula?
- Can you use the discriminant to identify the types and multiplicities of roots of a quadratic equation?
- Can you solve a system consisting of two linear equations in two variables using substitution, elimination, and the inverse of a matrix?
- Can you solve a system consisting of a linear equation and a quadratic equation in two variables?
- Can you find the intersection(s) of two curves algebraically or using technology?
- Can you calculate the average rate of change for functions?
- Can you calculate and interpret the intercepts and slope of a line?

C. Functions

1. Understand the function concept and the use of function notation
   a. recognize that functions are sets of ordered pairs
   b. understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range
   c. use function notation, evaluate functions, and interpret statements that use function notation in terms of a context
   d. recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers
2. Understand how to find the domain and range of a function and a relation
   a. identify the domain and range of a function or relation
   b. determine the domain of a function from a function rule (e.g., $f(x) = 2x + 1$), graph, set of ordered pairs, or table
3. Understand how function behavior is analyzed using different representations (e.g., graphs, mappings, tables)
   a. interpret key features of graphs and tables (e.g., increasing/decreasing, maximum/minimum, periodicity) in terms of the quantities for a function that models a relationship between two quantities
   b. given a verbal description of a relation, sketch graphs that show key features of that relation
   c. graph functions (i.e., radical, piecewise, absolute value, polynomial, rational, logarithmic, trigonometric) expressed symbolically and identify key features of the graph
   d. write a function that is defined by an expression in different but equivalent forms to reveal different properties of the function (e.g., zeros, extreme values, symmetry of the graph)
   e. interpret the behavior of exponential functions (e.g., growth, decay)
   f. understand how to determine if a function is odd, even, or neither, and any resulting symmetries
4. Understand how functions and relations are used to model relationships between quantities
   a. write a function that relates two quantities
   b. determine an explicit expression or a recursive process that builds a function from a context
5. Understand how new functions are obtained from existing functions (e.g., compositions, transformations, inverses)
   a. describe how the graph of $g(x)$ is related to the graph of $f(x)$, where $g(x) = f(x) + k$, $g(x) = k\,f(x)$, $g(x) = f(kx)$, or $g(x) = f(x + k)$ for specific values of k (both positive and negative), and find the value of k given the graphs
   b. determine if a function has an inverse and, if so, write an expression for the inverse
   c. verify by composition if one function is the inverse of another
   d. given that a function f has an inverse, find values of the inverse function from a graph or a table of f

e. given a noninvertible function, determine a largest possible domain of the function that produces an invertible function

f. understand the inverse relationship between exponential and logarithmic functions and use this relationship to solve problems

g. combine standard function types using arithmetic operations

h. perform domain analysis on functions resulting from arithmetic operations

i. compose functions algebraically, numerically, and graphically

j. perform domain analysis on functions resulting from compositions

6. Understand differences between linear, quadratic, and exponential models, including how their equations are created and used to solve problems

a. understand that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals

b. recognize situations in which one quantity changes at a constant rate per unit interval relative to another

c. recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another

d. construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two ordered pairs (including reading these from a table)

e. observe that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function

f. express the solution to an exponential equation with base *b* as a logarithm (e.g., $3 \cdot 2^{5t} = 20,\ 3 \cdot e^{5t} = 20$)

g. use technology to evaluate logarithms that have any base

h. interpret the parameters in a linear or exponential function in terms of a context (e.g., $A(t) = Pe^{rt}$)

i. use quantities that are inversely related to model phenomena

7. Understand how to construct the unit circle and how to use it to find values of trigonometric functions for all angle measures in their domains

a. understand radian measure (e.g., one radian is the measure of a central angle that subtends an arc with length equal to the length of the radius)

b. understand how the domains of trigonometric functions can be extended beyond 0 to 2π using the unit circle

c. use special triangles (i.e., 30-60-90, 45-45-90) to determine geometrically the values of sine, cosine, and tangent for $\frac{\pi}{3}$, $\frac{\pi}{4}$, and $\frac{\pi}{6}$

d. use reference angles to find the values of trigonometric functions at angles outside the interval 0 to $\frac{\pi}{2}$

e. use the unit circle to explain symmetry and periodicity of trigonometric functions

8. Understand how periodic phenomena are modeled using trigonometric functions

a. choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline

b. understand how to restrict the domain of a trigonometric function so that its inverse can be constructed

c. use inverse functions to solve trigonometric equations that arise in modeling contexts, and interpret them in terms of the context

9. Understand the application of trigonometric identities (e.g., Pythagorean, double angle, half angle, sum of angles, difference of angles)

a. use Pythagorean identities (e.g., $\sin^2\theta + \cos^2\theta = 1$)

b. use trigonometric identities to rewrite expressions and solve equations

c. understand trigonometric identities in the context of equivalent graphs of trigonometric functions (e.g., $y = \sin x$ and $y = \cos\left(\frac{\pi}{2} - x\right)$ are equivalent graphs)

d. prove Pythagorean identities (e.g., $\sin^2\theta + \cos^2\theta = 1$)

10. Know how to interpret representations of functions of two variables (e.g., three-dimensional graphs, tables)
    a. interpret representations of functions of two variables
11. Understand how to solve equations (e.g., trigonometric, logarithmic, exponential)
    a. solve trigonometric, logarithmic, and exponential equations

Discussion areas: Functions

- Can you recognize function notation and understand that for each input, the function produces one and only one output?
- Can you determine whether a relation is a function numerically, algebraically, and graphically?
- Can you recognize the domain as the set of valid inputs for a function and the range as the set of resulting outputs, and can you find these for a given function?
- Can you find the zeros, extreme values, intervals of increasing or decreasing, and symmetry of a function given a graph, algebraic representation, or verbal description?
- Can you graph radical, piecewise, absolute value, polynomial, rational, logarithmic, and trigonometric functions?
- Can you determine whether an exponential function will grow or decay and at what rate?
- Can you create a function that models a relationship between two described quantities?
- Can you take one or more functions and create another function using functional operations, function composition, and transformations?
- Can you identify the domain and range of the sum, product, difference, quotient, or composition of two functions?
- Can you find the inverse of a given function?
- Can you determine whether a function has an inverse?
- Can you determine whether two functions are inverses graphically and analytically?
- Can you determine the type of function (linear, quadratic, exponential) that best fits a given scenario or situation?
- Can you use the unit circle and special right triangles to determine the values of given trigonometric functions?
- Can you find the values of trigonometric functions for an angle given the value of one trigonometric function and the quadrant of the angle?
- Can you model situations that occur periodically using trigonometric functions?
- Can you recognize and use trigonometric identities?
- Can you identify and interpret three-dimensional graphs of two-variable functions?
- Can you express exponential equations as logarithms and evaluate logarithms with any base using technology?
- Can you solve logarithmic and trigonometric equations?

D. Calculus

1. Understand the meaning of a limit of a function and how to calculate limits of functions, determine when the limit does not exist, and solve problems using the properties of limits
   a. graphically analyze the limit of $f(x)$ as x approaches a fixed value from both left and right
   b. solve limit problems (e.g., a constant times a function, the sum of two functions, the product and quotient of two functions) using properties of limits, where all limits of the individual functions exist at the value that x is approaching
   c. analyze one-sided limits for various functions to see whether or not the limit exists
   d. recognize limits that do not exist, such as

$$\lim_{x\to 0} \sin\left(\frac{1}{x}\right) \text{ and } \lim_{x\to 0} \frac{1}{\sqrt[3]{x^2}}$$

2. Understand the derivative of a function as a limit, as the slope of a line tangent to a curve, and as a rate of change
    a. construct a function graph for a given function and a given point (a, $f(a)$), and explain what happens to the succession of slopes of secant lines connecting (a, $f(a)$) to (x, $f(x)$) as x approaches a, from both the right side and the left side
    b. state the limit definition of the derivative, and use it to find the derivative of a given function at a given value of x and to find the derivative function
3. Understand how to show that a particular function is continuous
    a. apply the three steps (i.e., $f(a)$ exists, $\lim_{x \to a} f(x)$ exists, and $f(a) = \lim_{x \to a} f(x)$) that are part of the definition of what it means for a function to be continuous at $x = a$ to verify whether a given function is continuous at a given point
4. Know the relationship between continuity and differentiability
    a. give examples of functions that are continuous at $x = a$ but not differentiable at $x = a$, and explain why
5. Understand how to approximate derivatives and integrals numerically
    a. given a table of values, use the slope of a secant line to approximate a derivative
    b. use the midpoint rule, trapezoid rule, or other Reimann sums to find numerical approximations for integrals
6. Understand how and when to use standard differentiation and integration techniques
    a. use standard differentiation techniques
    b. use standard integration techniques
    c. understand the relationship between position, velocity, and acceleration functions of a particle in motion
7. Understand how to analyze the behavior of a function (e.g., extrema, concavity, symmetry)
    a. use the first and second derivatives to analyze the graph of a function
8. Understand how to apply derivatives to solve problems (e.g., related rates, optimization)
    a. apply derivatives to solve problems
9. Understand the foundational theorems of calculus (e.g., fundamental theorems of calculus, mean value theorem, intermediate value theorem)
    a. solve problems using the foundational theorems of calculus
    b. understand the relationship between differentiation and integration, including the role of the fundamental theorems of calculus
    c. match graphs of functions with graphs of their derivatives or accumulations
    d. understand how to use differentiation and integration of a function to express rates of change and total change
    e. understand and calculate the average value of a function over an interval (i.e., mean value theorem of integrals)
10. Understand integration as a limit of Riemann sums
    a. calculate a definite integral using a limit of Riemann sums
11. Understand how to use integration to compute area, volume, distance, or other accumulation processes
    a. use integration techniques to compute area, volume, distance, or other accumulation processes
12. Know how to determine the limits of sequences, if they exist
    a. determine the limits of sequences, when they exist
13. Is familiar with simple infinite series
    a. determine if simple infinite series converge or diverge
    b. find the sum of a simple infinite series, if it exists
    c. find the partial sum of a simple infinite series

Discussion areas: Calculus

- Can you compute and analyze the limit of a function at a given point?
- Can you recognize the difference between the limit of a function at a point and the value of the function at the point?
- Can you use the limit definition of the derivative to differentiate a function?
- Can you approximate derivatives given a table by using the slope of a secant line?
- Can you determine the continuity and differentiability of a function at a given point?
- Can you calculate the extrema of a function and determine when the function is increasing or decreasing using the first derivatives?
- Can you find the concavity of a curve at a particular point and find any point(s) of inflection?
- Can you apply derivatives to solve related rate of optimization problems?
- Can you approximate integrals by using the midpoint rule, trapezoidal rule, or other Riemann sums?
- Can you compute a definite integral using the fundamental theorem of calculus?
- Can you use integration techniques to calculate accumulated change from a rate of change?
- Can you use differentiation and integration techniques to identify the relationship between position, velocity, and acceleration functions?
- Can you test for convergence or divergence of a simple series?
- Can you calculate the sum, if it exists, and partial sums of simple infinite series?

II. Geometry, Probability and Statistics, and Discrete Mathematics

A. Geometry

1. Understand transformations in a plane
   a. know precise definitions of angle, circle, line segment, perpendicular lines, and parallel lines
   b. represent transformations in the plane
   c. describe transformations as functions that take points in the plane as inputs, and give other points as outputs
   d. recognize whether a transformation preserves distance and angle measure
   e. given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that map it onto itself
   f. develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments
   g. given a geometric figure and a rotation, reflection, or translation, draw the transformed figure
   h. specify a sequence of transformations that will map a given figure onto another figure
2. Understand how to prove geometric theorems, such as those about lines and angles, triangles, and parallelograms
   a. prove theorems about lines and angles
   b. prove theorems about triangles
   c. prove theorems about parallelograms
3. Understand how geometric constructions are made with a variety of tools and methods
   a. recognize formal geometric constructions
   b. explain how formal geometric constructions are made (e.g., an equilateral triangle, a square, a regular hexagon inscribed in a circle)
4. Understand congruence and similarity in terms of transformations
   a. use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure
   b. verify the properties of dilations given by a center and a scale factor

   c. given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent
   d. given two figures, use the definition of similarity in terms of dilations to decide if the figures are similar
   e. explain how the criteria for triangle congruence (e.g., ASA, SAS, HL) follow from the definition of congruence in terms of rigid motions
   f. use the properties of similarity transformations to establish the AA criterion for two triangles to be similar
   g. use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures

5. Understand how trigonometric ratios are defined in right triangles
   a. understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles
   b. explain and use the relationship between the sine and cosine of complementary angles
   c. use trigonometric ratios and the Pythagorean theorem to solve right triangles in applied problems
6. Understand how trigonometry is applied to general triangles
   a. derive the formula $A = \frac{1}{2}ab\sin C$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side and use it to solve problems
   b. apply the law of sines and the law of cosines to find unknown measurements in triangles
7. Understand and applies theorems about circles
   a. identify and describe relationships among inscribed angles, radii, and chords
   b. prove properties of angles for a quadrilateral inscribed in a circle
   c. construct a tangent line from a point outside a given circle to the circle
8. Understand arc length and area measurements of sectors of circles
   a. derive and use the fact that the length of the arc intercepted by a central angle is proportional to the circumference
   b. derive and use the formula for the area of a sector
9. Know how to translate between a geometric description (e.g., focus, asymptotes, directrix) and an equation for a conic section
   a. derive and use the equation of a circle of given center and radius
   b. complete the square to find the center and radius of a circle given by an equation in standard form
   c. derive the equation of a parabola given a focus and directrix
   d. derive and use the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from a point on the curve to the foci is constant
10. Understand how to use coordinate geometry to algebraically prove simple geometric theorems
   a. use coordinates to prove simple geometric theorems algebraically
   b. prove the slope criteria for parallel and perpendicular lines, and use parallel and perpendicular lines to solve geometric problems
   c. find the point on a directed line segment between two given points that partitions the segment in a given ratio
   d. use coordinates to compute perimeters of polygons and areas of triangles and quadrilaterals
11. Understand how perimeter, area, surface area, and volume formulas are used to solve problems
   a. give an informal argument for the formulas for the circumference of a circle, the area of a circle, and the volume of a cylinder, pyramid, and cone
   b. use the perimeter and area of geometric shapes to solve problems
   c. use the surface area and volume of prisms, cylinders, pyramids, cones, and spheres to solve problems

12. Know how to visualize relationships (e.g., cross section, nets, rotations) between two-dimensional and three-dimensional objects
    a. identify the shapes of two-dimensional cross sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects
    b. use two-dimensional representations of three-dimensional objects to visualize and solve problems
13. Know how to apply geometric concepts in real-world situations
    a. use geometric shapes, their measures, and their properties to describe objects
    b. apply concepts of density based on area and volume in modeling situations
    c. apply geometric methods to solve design problems
14. Understand the properties of parallel and perpendicular lines, triangles, quadrilaterals, polygons, and circles and how they can be used in problem solving
    a. solve problems involving parallel, perpendicular, and intersecting lines
    b. apply angle relationships (e.g., supplementary, vertical, alternate interior) to solve problems
    c. solve problems that involve medians, midpoints, and altitudes
    d. solve problems involving special triangles (e.g., isosceles, equilateral, right)
    e. know geometric properties of various quadrilaterals (e.g., parallelograms, trapezoids)
    f. know relationships among quadrilaterals
    g. solve problems involving angles and diagonals
    h. solve problems involving polygons with more than four sides

Discussion areas: Geometry

- Can you use the definitions of angles, circles, line segments, perpendicular lines, and parallel lines?
- Can you translate, reflect, rotate, and dilate figures and describe these transformations as functions?
- Can you use the relationships of the angles formed when parallel lines are cut by a transversal?
- Can you prove and apply the theorems of supplementary angles, complementary angles, vertical angles, exterior angles, triangle sum, and base angles?
- Can you construct geometric figures using a straight edge and compass?
- Can you apply triangle congruence and similarity criteria to solve problems?
- Can you describe some real-life applications that involve the Pythagorean theorem and trigonometric ratios?
- Can you use the law of sines and the law of cosines to solve problems?
- Can you use the definitions, properties, and theorems about circles, such as angles, radii, chords, arcs, and tangents?
- Can you derive and use the formula for the arc length and the sector area of a circle?
- Can you derive the equations of a circle, a parabola, an ellipses, and a hyperbola given some of the geometric descriptions, such as focus, asymptotes, and directrix?
- Can you compute the perimeter of a polygon and the area of a triangle and a quadrilateral using coordinates?
- Can you apply the correct formula to compute the surface area and volume of prisms, cylinders, pyramids, cones, and spheres?
- Can you identify 2-dimensional cross sections of 3-dimensional shapes?
- Can you use 2-dimensional (nets) to represent 3-dimensional objects?
- Can you use the definitions and properties of special triangles (e.g., isosceles, equilateral, right) and special polygons (e.g., regular polygon, parallelogram, trapezoids)?

B. Probability and Statistics

1. Understand how to summarize, represent, and interpret data collected from measurements on a single variable (e.g., box plots, dot plots, normal distributions)
   a. represent data with plots on the real number line (e.g., dot plots, histograms, and box plots)
   b. use statistics appropriate to the shape of the data distribution to compare center (e.g., median, mean) and spread (e.g., interquartile range, standard deviation) of two or more different data sets
   c. interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of outliers
   d. use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages, and recognize that there are data sets for which such a procedure is not appropriate
   e. estimate areas under the normal curve
2. Understand how to summarize, represent, and interpret data collected from measurements on two variables, either categorical or quantitative (e.g., scatterplots, time series)
   a. summarize and interpret categorical data for two categories in two-way frequency tables (e.g., joint, marginal, conditional relative frequencies)
   b. recognize possible associations and trends in the data
   c. represent data for two quantitative variables on a scatterplot, and describe how the variables are related
3. Understand how to create and interpret linear regression models (e.g., rate of change, intercepts, correlation coefficient)
   a. use technology to fit a function to data (i.e., linear regression)
   b. use functions fitted to data to solve problems in the context of the data
   c. assess the fit of a function by plotting and analyzing residuals
   d. interpret the slope and the intercept of a regression line in the context of the data
   e. compute and interpret a correlation coefficient
   f. distinguish between correlation and causation
4. Understand statistical processes and how to evaluate them
   a. understand statistics as a process for making inferences about population parameters based on a random sample from that population
   b. decide if a specified model is consistent with results from a given data-generating process (e.g., using simulation)
5. Understand how to make inferences and justify conclusions from samples, experiments, and observational studies
   a. recognize the purposes of and differences among samples, experiments, and observational studies, and explain how randomization relates to each
   b. use data from a sample to estimate a population mean or proportion
   c. use data from a randomized experiment to compare two treatments
   d. use results of simulations to decide if differences between parameters are significant
   e. evaluate reports based on data
6. Understand the concepts of independence and conditional probability and how to apply these concepts to data
   a. describe events as subsets of a sample space using characteristics of the outcomes, or as unions, intersections, or complements of other events
   b. understand that two events, *A* and *B*, are independent if and only if $P(A \cap B) = P(A)P(B)$
   c. understand the conditional probability of *A* given *B* as $\frac{P(A \text{ and } B)}{P(B)}$, and interpret independence of *A* and *B* as saying that $P(A|B) = P(A)$ and $P(B \mid A) = P(B)$
   d. recognize and explain the concepts of conditional probability and independence

7. Understand how to compute probabilities of simple events, probabilities of compound events, and conditional probabilities
   a. calculate probabilities of simple and compound events
   b. construct and interpret two-way frequency tables of data when two categories are associated with each object being classified; use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities
   c. find $P(A|B)$, and interpret it in terms of a given model
   d. apply the addition rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret it in terms of a given model
   e. apply the general multiplication rule in a uniform probability model, $P(A \text{ and } B) = P(A)P(B|A) = P(B)P(A|B)$, and interpret it in terms of a given model
   f. calculate probabilities using the binomial probability distribution
8. Know how to make informed decisions using probabilities and expected values
   a. define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space, and graph the corresponding probability distribution using the same graphical displays as for data distributions
   b. calculate the expected value of a random variable, and interpret it as the mean of the probability distribution
   c. develop a probability distribution for a random variable, defined for a sample space in which theoretical probabilities can be calculated, and find the expected value
   d. develop a probability distribution for a random variable, defined for a sample space in which probabilities are assigned empirically, and find the expected value
   e. weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values
   f. analyze decisions and strategies using probability concepts (e.g., fairness)
9. Understand how to use simulations to construct experimental probability distributions and to make informal inferences about theoretical probability distributions
   a. given the results of simulations, construct experimental probability distributions
   b. given the results of simulations, make informal inferences about theoretical probability distributions
10. Understand how to find probabilities involving finite sample spaces and independent trials
    a. use the fundamental counting principle to find probabilities involving finite sample spaces and independent trials

Discussion areas: Probability and Statistics

- Can you determine measures of center and spread for single-variable data presented in a variety of formats?
- Can you determine the differences between mean, median, and mode, including advantages and disadvantages of each?
- Can you analyze data presented in scatterplots and use this to predict associations or trends between two variables?
- Can you construct and interpret two-way frequency tables?
- Can you calculate the correlation coefficient between two variables and discuss the possibility of causation, causation by a third event, and coincidence?
- Can you compute and interpret the meaning of a confidence interval for a population proportion or mean given results of a sample?
- Can you calculate conditional probabilities and understand the idea of independent events?
- Can you compute the possibility of a single outcome occurring, one of multiple outcomes occurring, and an outcome occurring given certain conditions?
- Can you determine the expected gain or loss in a game of chance?

C. Discrete Mathematics

1. Understand sequences (e.g., arithmetic, recursively defined, geometric)
    a. write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms
    b. evaluate, extend, or algebraically represent rules that involve number patterns
    c. explore patterns in order to make conjectures, predictions, or generalizations
2. Is familiar with how recursion can be used to model various phenomena
    a. find values of functions defined recursively, and understand how recursion can be used to model various phenomena
    b. convert between recursive and closed-form expressions for a function, where possible
3. Has knowledge of equivalence relations
    a. determine whether a binary relation on a set is reflexive, symmetric, or transitive
    b. determine whether a relation is an equivalence relation
4. Understand the differences between discrete and continuous representations (e.g., data, functions) and how each can be used to model various phenomena
    a. understand the differences between discrete and continuous representations (e.g., data, functions)
    b. understand how discrete and continuous representations can be used to model various phenomena
5. Understand basic terminology and symbols of logic
    a. understand the basic terminology of logic
    b. understand the symbols of logic
    c. use logic to evaluate the truth of statements
    d. use logic to evaluate the equivalence of statements (e.g., statement and contrapositive)
6. Understand how to use counting techniques such as the multiplication principle, permutations, and combinations
    a. use counting techniques to solve problems
7. Understand basic set theory (e.g., unions, differences, Venn diagrams)
    a. solve problems using basic set theory (i.e., union, intersection, complement, difference)
    b. use Venn diagrams to answer questions about sets

Discussion areas: Discrete Mathematics

- Can you recognize and define sequences as recursive or explicit functions?
- Can you determine whether a binary relation on a set is reflexive, symmetric, or transitive?
- Can you explain the difference between discrete and continuous data or functions and determine which is best to use as a model given a scenario or situation?
- Can you recognize and use terminology and symbols of logic to evaluate the truth or equivalence of statements?
- Can you recognize logical fallacies such as assuming the equivalence of a proposition and its converse?
- Can you use counting techniques such as permutations and combinations to determine the number of outcomes in a given scenario or situation?
- Can you analyze and interpret unions, intersections, complements, and differences of sets given descriptions and/or Venn diagrams?

Mathematical Process Categories

Mathematical Problem Solving

- Solve problems that arise in mathematics and those involving mathematics in other contexts
- Build new mathematical knowledge through problem solving
- Apply and adapt a variety of appropriate strategies

Mathematical Reasoning and Proof

- Select and use various types of reasoning and methods of proof
- Make and investigate mathematical conjectures
- Develop and evaluate mathematical arguments and proofs

Mathematical Connections

- Recognize and use connections among mathematical ideas
- Apply mathematics in context outside of mathematics
- Demonstrate an understanding of how mathematical ideas interconnect and build on one another

Mathematical Representation

- Select, apply, and translate among mathematical representations to solve problems
- Use representations to model and interpret physical, social, and mathematical phenomena
- Create and use representations to organize, record, and communicate mathematical ideas

Use of Technology

- Use technology as an aid to understanding mathematical ideas
- Use technology appropriately as a tool for problem solving

7. Review Smart Tips for Success

Follow test-taking tips developed by experts

Learn from the experts. Take advantage of the following answers to questions you may have and practical tips to help you navigate the *Praxis* test and make the best use of your time.

Should I guess?

Yes. Your score is based on the number of questions you answer correctly, with no penalty or subtraction for an incorrect answer. When you don't know the answer to a question, try to eliminate any obviously wrong answers and then guess at the correct one. Try to pace yourself so that you have enough time to carefully consider every question.

Can I answer the questions in any order?

You can answer the questions in order or skip questions and come back to them later. If you skip a question, you can also mark it so that you can remember to return and answer it later. Remember that questions left unanswered are treated the same as questions answered incorrectly, so it is to your advantage to answer every question.

Are there trick questions on the test?

No. There are no hidden meanings or trick questions. All of the questions on the test ask about subject matter knowledge in a straightforward manner.

Are there answer patterns on the test?

No. You might have heard this myth: the answers on tests follow patterns. Another myth is that there will never be more than two questions in a row with the correct answer in the same position among the choices. Neither myth is true. Select the answer you think is correct based on your knowledge of the subject.

Can I write on the scratch paper I am given?

Yes. You can work out problems on the scratch paper, make notes to yourself, or write anything at all. Your scratch paper will be destroyed after you are finished with it, so use it in any way that is helpful to you. But make sure to select or enter your answers on the computer.

Smart Tips for Taking the Test

1. **Skip the questions you find extremely difficult.** Rather than trying to answer these on your first pass through the test, you may want to leave them blank and mark them so that you can return to them later. Pay attention to the time as you answer the rest of the questions on the test, and try to finish with 10 or 15 minutes remaining so that you can go back over the questions you left blank. Even if you don't know the answer the second time you read the questions, see if you can narrow down the possible answers, and then guess. Your score is based on the number of right answers, so it is to your advantage to answer every question.

2. **Keep track of the time.** The on-screen clock will tell you how much time you have left. You will probably have plenty of time to answer all of the questions, but if you find yourself becoming bogged down, you might decide to move on and come back to any unanswered questions later.

3. **Read all of the possible answers before selecting one.** For questions that require you to select more than one answer, or to make another kind of selection, consider the most likely answers given what the question is asking. Then reread the question to be sure the answer(s) you have given really answer the question. Remember, a question that contains a phrase such as "Which of the following does NOT …" is asking for the one answer that is NOT a correct statement or conclusion.

4. **Check your answers.** If you have extra time left over at the end of the test, look over each question and make sure that you have answered it as you intended. Many test takers make careless mistakes that they could have corrected if they had checked their answers.

5. **Don't worry about your score when you are taking the test.** No one is expected to answer all of the questions correctly. Your score on this test is not analogous to your score on the *GRE*® or other tests. It doesn't matter on the *Praxis* tests whether you score very high or barely pass. If you meet the minimum passing scores for your state and you meet the state's other requirements for obtaining a teaching license, you will receive a license. In other words, what matters is meeting the minimum passing score. You can find passing scores for all states that use the *Praxis* tests at http://www.ets.org/s/praxis/pdf/passing_scores.pdf or on the web site of the state for which you are seeking certification/licensure.

6. **Use your energy to take the test, not to get frustrated by it.** Getting frustrated only increases stress and decreases the likelihood that you will do your best. Highly qualified educators and test development professionals, all with backgrounds in teaching, worked diligently to make the test a fair and valid measure of your knowledge and skills. Your state painstakingly reviewed the test before adopting it as a licensure requirement. The best thing to do is concentrate on answering the questions.

8. Check on Testing Accommodations

See if you qualify for accommodations that may make it easier to take the Praxis *test*

What if English is not my primary language?

Praxis tests are given only in English. If your primary language is not English (PLNE), you may be eligible for extended testing time. For more details, visit www.ets.org/praxis/register/plne_accommodations/.

What if I have a disability or other health-related need?

The following accommodations are available for *Praxis* test takers who meet the Americans with Disabilities Act (ADA) Amendments Act disability requirements:

- Extended testing time
- Additional rest breaks
- Separate testing room
- Writer/recorder of answers
- Test reader
- Sign language interpreter for spoken directions only
- Perkins Brailler
- Braille slate and stylus
- Printed copy of spoken directions
- Oral interpreter
- Audio test
- Braille test
- Large print test book
- Large print answer sheet
- Listening section omitted

For more information on these accommodations, visit www.ets.org/praxis/register/disabilities.

Note: Test takers who have health-related needs requiring them to bring equipment, beverages, or snacks into the testing room or to take extra or extended breaks must request these accommodations by following the procedures described in the *Bulletin Supplement for Test Takers with Disabilities or Health-Related Needs* (PDF), which can be found at http://www.ets.org/s/disabilities/pdf/bulletin_supplement_test_takers_with_disabilities_health_needs.pdf.

You can find additional information on available resources for test takers with disabilities or health-related needs at www.ets.org/disabilities.

9. Do Your Best on Test Day

Get ready for test day so you will be calm and confident

You followed your study plan. You prepared for the test. Now it's time to prepare for test day.

Plan to end your review a day or two before the actual test date so you avoid cramming. Take a dry run to the test center so you're sure of the route, traffic conditions, and parking. Most of all, you want to eliminate any unexpected factors that could distract you from your ultimate goal—passing the *Praxis* test!

On the day of the test, you should:

- be well rested
- wear comfortable clothes and dress in layers
- eat before you take the test
- bring an acceptable and valid photo identification with you
- bring an approved calculator only if one is specifically permitted for the test you are taking (see Calculator Use, at http://www.ets.org/praxis/test_day/policies/calculators)
- be prepared to stand in line to check in or to wait while other test takers check in

You can't control the testing situation, but you can control yourself. Stay calm. The supervisors are well trained and make every effort to provide uniform testing conditions, but don't let it bother you if the test doesn't start exactly on time. You will have the allotted amount of time once it does start.

You can think of preparing for this test as training for an athletic event. Once you've trained, prepared, and rested, give it everything you've got.

What items am I restricted from bringing into the test center?

You cannot bring into the test center personal items such as:

- handbags, knapsacks, or briefcases
- water bottles or canned or bottled beverages
- study materials, books, or notes
- pens, pencils, scrap paper, or calculators, unless specifically permitted for the test you are taking (see Calculator Use, at http://www.ets.org/praxis/test_day/policies/calculators)
- any electronic, photographic, recording, or listening devices

Personal items are not allowed in the testing room and will not be available to you during the test or during breaks. You may also be asked to empty your pockets. At some centers, you will be assigned a space to store your belongings, such as handbags and study materials. Some centers do not have secure storage space available, so please plan accordingly.

Test centers assume no responsibility for your personal items.

If you have health-related needs requiring you to bring equipment, beverages or snacks into the testing room or to take extra or extended breaks, you need to request accommodations in advance. Procedures for requesting accommodations are described in the Bulletin Supplement for Test Takers with Disabilities or Health-related Needs (PDF).

Note: All cell phones, smart phones (e.g., Android® devices, iPhones®, etc.), and other electronic, photographic, recording, or listening devices are strictly prohibited from the test center. If you are seen with such a device, you will be dismissed from the test, your test scores will be canceled, and you will forfeit your test fees. If you are seen *using* such a device, the device will be confiscated and inspected. For more information on what you can bring to the test center, visit www.ets.org/praxis/test_day/bring.

Are You Ready?

Complete this checklist to determine whether you are ready to take your test.

- ❒ Do you know the testing requirements for the license or certification you are seeking in the state(s) where you plan to teach?
- ❒ Have you followed all of the test registration procedures?
- ❒ Do you know the topics that will be covered in each test you plan to take?
- ❒ Have you reviewed any textbooks, class notes, and course readings that relate to the topics covered?
- ❒ Do you know how long the test will take and the number of questions it contains?
- ❒ Have you considered how you will pace your work?
- ❒ Are you familiar with the types of questions for your test?
- ❒ Are you familiar with the recommended test-taking strategies?
- ❒ Have you practiced by working through the practice questions in this study companion or in a study guide or practice test?
- ❒ If constructed-response questions are part of your test, do you understand the scoring criteria for these questions?
- ❒ If you are repeating a *Praxis* test, have you analyzed your previous score report to determine areas where additional study and test preparation could be useful?

If you answered "yes" to the questions above, your preparation has paid off. Now take the *Praxis* test, do your best, pass it—and begin your teaching career!

10. Understand Your Scores

Understand how tests are scored and how to interpret your test scores

Of course, passing the *Praxis* test is important to you so you need to understand what your scores mean and what your state requirements are.

What are the score requirements for my state?

States, institutions, and associations that require the tests set their own passing scores. Visit www.ets.org/praxis/states for the most up-to-date information.

If I move to another state, will my new state accept my scores?

The *Praxis* tests are part of a national testing program, meaning that they are required in many states for licensure. The advantage of a national program is that if you move to another state that also requires *Praxis* tests, you can transfer your scores. Each state has specific test requirements and passing scores, which you can find at www.ets.org/praxis/states.

How do I know whether I passed the test?

Your score report will include information on passing scores for the states you identified as recipients of your test results. If you test in a state with automatic score reporting, you will also receive passing score information for that state.

A list of states and their passing scores for each test are available online at www.ets.org/praxis/states.

What your *Praxis* scores mean

You received your score report. Now what does it mean? It's important to interpret your score report correctly and to know what to do if you have questions about your scores.

Visit http://www.ets.org/s/praxis/pdf/sample_score_report.pdf to see a sample score report.
To access *Understanding Your Praxis Scores*, a document that provides additional information on how to read your score report, visit www.ets.org/praxis/scores/understand.

Put your scores in perspective

Your score report indicates:

- Your score and whether you passed
- The range of possible scores
- The raw points available in each content category
- The range of the middle 50 percent of scores on the test

If you have taken the same *Praxis* test or other *Praxis* tests in the last 10 years, your score report also lists the highest score you earned on each test taken.

Content category scores and score interpretation

Questions on the *Praxis* tests are categorized by content. To help you in future study or in preparing to retake the test, your score report shows how many raw points you earned in each content category. Compare your "raw points earned" with the maximum points you could have earned ("raw points available"). The greater the difference, the greater the opportunity to improve your score by further study.

Score scale changes

ETS updates *Praxis* tests on a regular basis to ensure they accurately measure the knowledge and skills that are required for licensure. When tests are updated, the meaning of the score scale may change, so requirements may vary between the new and previous versions. All scores for previous, discontinued tests are valid and reportable for 10 years, provided that your state or licensing agency still accepts them.

These resources may also help you interpret your scores:

- *Understanding Your Praxis Scores* (PDF), found at www.ets.org/praxis/scores/understand
- *The Praxis Passing Scores* (PDF), found at www.ets.org/praxis/scores/understand
- State requirements, found at www.ets.org/praxis/states

Appendix: Other Questions You May Have

Here is some supplemental information that can give you a better understanding of the *Praxis* tests.

What do the *Praxis* tests measure?

The *Praxis* tests measure the specific knowledge and skills that beginning teachers need. The tests do not measure an individual's disposition toward teaching or potential for success, nor do they measure your actual teaching ability. The assessments are designed to be comprehensive and inclusive but are limited to what can be covered in a finite number of questions and question types. Teaching requires many complex skills that are typically measured in other ways, including classroom observation, video recordings, and portfolios.

Ranging from Agriculture to World Languages, there are more than 80 *Praxis* tests, which contain selected-response questions or constructed-response questions, or a combination of both.

Who takes the tests and why?

Some colleges and universities use the *Praxis* Core Academic Skills for Educators tests (Reading, Writing, and Mathematics) to evaluate individuals for entry into teacher education programs. The assessments are generally taken early in your college career. Many states also require Core Academic Skills test scores as part of their teacher licensing process.

Individuals entering the teaching profession take the *Praxis* content and pedagogy tests as part of the teacher licensing and certification process required by many states. In addition, some professional associations and organizations require *Praxis* Subject Assessments for professional licensing.

Do all states require these tests?

The *Praxis* tests are currently required for teacher licensure in approximately 40 states and United States territories. These tests are also used by several professional licensing agencies and by several hundred colleges and universities. Teacher candidates can test in one state and submit their scores in any other state that requires *Praxis* testing for licensure. You can find details at www.ets.org/praxis/states.

What is licensure/certification?

Licensure in any area—medicine, law, architecture, accounting, cosmetology—is an assurance to the public that the person holding the license possesses sufficient knowledge and skills to perform important occupational activities safely and effectively. In the case of teacher licensing, a license tells the public that the individual has met predefined competency standards for beginning teaching practice.

Because a license makes such a serious claim about its holder, licensure tests are usually quite demanding. In some fields, licensure tests have more than one part and last for more than one day. Candidates for licensure in all fields plan intensive study as part of their professional preparation. Some join study groups, others study alone. But preparing to take a licensure test is, in all cases, a professional activity. Because a licensure exam surveys a broad body of knowledge, preparing for a licensure exam takes planning, discipline, and sustained effort.

Why does my state require the *Praxis* tests?

Your state chose the *Praxis* tests because they assess the breadth and depth of content—called the "domain"—that your state wants its teachers to possess before they begin to teach. The level of content knowledge, reflected in the passing score, is based on recommendations of panels of teachers and teacher educators in

each subject area. The state licensing agency and, in some states, the state legislature ratify the passing scores that have been recommended by panels of teachers.

How were the tests developed?

ETS consulted with practicing teachers and teacher educators around the country during every step of the *Praxis* test development process. First, ETS asked them what knowledge and skills a beginning teacher needs to be effective. Their responses were then ranked in order of importance and reviewed by hundreds of teachers.

After the results were analyzed and consensus was reached, guidelines, or specifications, for the selected-response and constructed-response tests were developed by teachers and teacher educators. Following these guidelines, teachers and professional test developers created test questions that met content requirements and *ETS Standards for Quality and Fairness*.*

When your state adopted the research-based *Praxis* tests, local panels of teachers and teacher educators evaluated each question for its relevance to beginning teachers in your state. During this "validity study," the panel also provided a passing-score recommendation based on how many of the test questions a beginning teacher in your state would be able to answer correctly. Your state's licensing agency determined the final passing-score requirement.

ETS follows well-established industry procedures and standards designed to ensure that the tests measure what they are intended to measure. When you pass the *Praxis* tests your state requires, you are proving that you have the knowledge and skills you need to begin your teaching career.

How are the tests updated to ensure the content remains current?

The *Praxis* tests are reviewed regularly. During the first phase of review, ETS conducts an analysis of relevant state and association standards and of the current test content. State licensure titles and the results of relevant job analyses are also considered. Revised test questions are then produced following the standard test development methodology. National advisory committees may also be convened to review and revise existing test specifications and to evaluate test forms for alignment with the specifications.

How long will it take to receive my scores?

Scores for tests that do not include constructed-response questions are available on screen immediately after the test. Scores for tests that contain constructed-response questions or essays aren't available immediately after the test because of the scoring process involved. Official score reports are available to you and your designated score recipients approximately two to three weeks after the test date for tests delivered continuously, or two to three weeks after the testing window closes for other tests. See the test dates and deadlines calendar at www.ets.org/praxis/register/centers_dates for exact score reporting dates.

Can I access my scores on the web?

All test takers can access their test scores via My *Praxis* Account free of charge for one year from the posting date. This online access replaces the mailing of a paper score report.

The process is easy—simply log into My *Praxis* Account at www.ets.org/praxis and click on your score report. If you do not already have a *Praxis* account, you must create one to view your scores.

Note: You must create a *Praxis* account to access your scores, even if you registered by mail or phone.

**ETS Standards for Quality and Fairness* (2014, Princeton, N.J.) are consistent with the *Standards for Educational and Psychological Testing*, industry standards issued jointly by the American Educational Research Association, the American Psychological Association, and the National Council on Measurement in Education (2014, Washington, D.C.).

Your teaching career is worth preparing for, so start today!

Let the *Praxis*® *Study Companion* guide you.

To search for the *Praxis* test prep resources that meet your specific needs, visit:

www.ets.org/praxis/testprep

To purchase official test prep made by the creators of the *Praxis* tests, visit the ETS Store:

www.ets.org/praxis/store

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